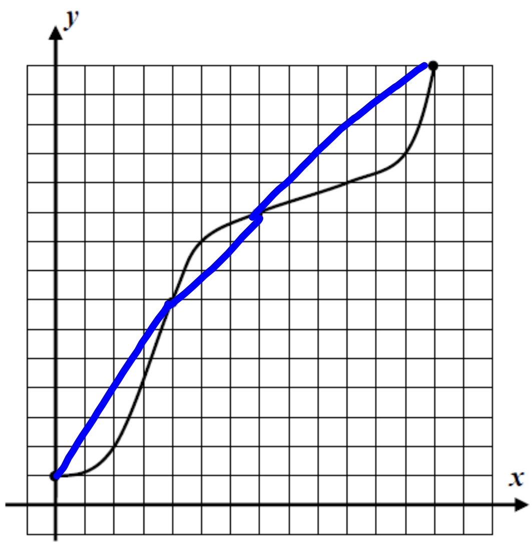
Average Rate of Change
Common Core Algebra II

When we model using functions, we are very often interested in the rate that the output is changing compared to the rate of the input.

***Exercise* #1:** The function $f(x)$ is shown graphed to the right.



(a) Evaluate each of the following based on the graph:

(i) $f(0)$ 1 (ii) $f(4)$ 7 (iii) $f(7)$ 10 (iv) $f(13)$ 15

(b) Find the change in the function, Δf, over each of the following domain intervals. Find this both by subtraction and show this on the graph.

(i) $0 \le x \le 4$ 6 (ii) $4 \le x \le 7$ 3 (iii) $7 \le x \le 13$ 5

(c) Why can't you simply compare the changes in f from part (b) to determine over which interval the function changing the fastest?

Because they have different x-intervals

(d) Calculate the **average rate of change** for the function over each of the intervals and determine which interval has the greatest rate.

(i) $0 \le x \le 4$

$\frac{6}{4} = 1.5$

(ii) $4 \le x \le 7$

$\frac{3}{3} = 1$

(iii) $7 \le x \le 13$

$\frac{5}{6} = .8\bar{3}$

(e) Using a straightedge, draw in the lines whose slopes you found in part (d) by connecting the points shown on the graph. The average rate of change gives a measurement of what property of the line?

slope

The average rate of change is an exceptionally important concept in mathematics because it gives us a way to **quantify** how fast a function changes on average over a certain **domain interval**. Although we used its formula in the last exercise, we state it formally here:

AVERAGE RATE OF CHANGE

For a function over the domain interval $a \le x \le b$, the function's **average rate of change** is calculated by:

$$\frac{\Delta f}{\Delta x} = \frac{\text{change in the output}}{\text{change in the input}} = \frac{f(b) - f(a)}{b - a}$$

***Exercise* #2:** Consider the two functions $f(x)=5x+7$ and $g(x)=2x^2+1$.

(a) Calculate the average rate of change for both functions over the following intervals. Do your work carefully and show the calculations that lead to your answers.

(i) $-2 \le x \le 3$

$f(-2) = 5(-2)+7 = -3$
$f(3) = 5(3)+7 = 22$
$R \text{ of } C = \frac{22-(-3)}{3-(-2)} = \frac{25}{5} = 5$
$g(-2) = 2(-2)^2+1 = 9$
$g(3) = 2(3)^2+1 = 19$
$R \text{ of } C = \frac{19-9}{3-(-2)} = \frac{10}{5} = 2$

(ii) $1 \le x \le 5$

$f(1) = 5(1)+7 = 12$
$f(5) = 5(5)+7 = 32$
$R \text{ of } C = \frac{32-12}{5-1} = \frac{20}{4} = 5$
$g(1) = 2(1)^2+1 = 3$
$g(5) = 2(5)^2+1 = 51$
$R \text{ of } C = \frac{51-3}{5-1} = \frac{48}{4} = 12$

(b) The average rate of change for *f* was the same for both (i) and (ii) but was not the same for *g*. Why is that?

f is linear so has constant Rate of change.
g is not linear

***Exercise* #3:** The table below represents a linear function. Fill in the missing entries.

+6 +8 +26

x	1	5	11	19	45
y	-5	1	10	22	61

+9 +12 +39

$y = mx + b$

$m = \frac{3}{2}$

$\frac{\Delta y}{\Delta x} = \frac{1-(-5)}{5-1} = \frac{6}{4} = \frac{3}{2} = \frac{9}{6}$

$\frac{3}{2} = \frac{12}{8}$

$\frac{3}{2} = \frac{39}{26}$