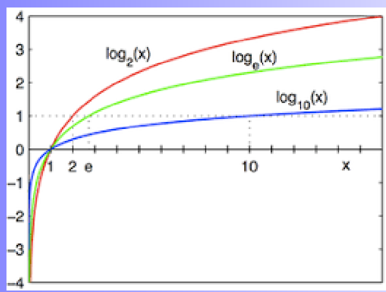
Introduction to Logarithms

Index = Logarithm

$N = a^x$ $\qquad$ $\log_a N = x$

Index form $\qquad$ Logarithm form



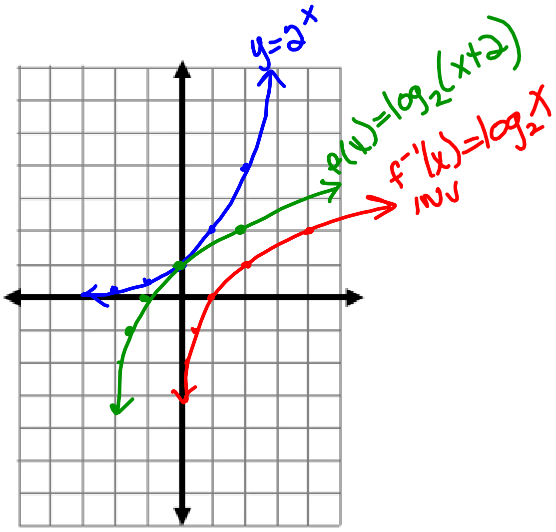
Introduction to Logarithms Unit 10 Day 1

Given the following function, make a table & graph $f(x) = 2^x$ and its inverse.

$f(x) = 2^x$ Inverse: switch x & y

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4

x	y
1/4	-2
1/2	-1
1	0
2	1
4	2



Is $f(x) = 2^x$ a one-to-one function? Explain your answer.

yes b/c it passes the horizontal and vertical line tests

Based on your answer, what must be true about the inverse of this function?

The inverse is also a function (1 to 1)

Based on the graph of the inverse of f(x) above,

Describe the end behavior of the ~~function~~ inv as $x \to \infty$ $y \to \infty$

Describe the end behavior of the ~~function~~ inv as $x \to 0$ $y \to -\infty$

What would be the first two steps to find an equation for the inverse of f(x) = 2^x algebraically?

1 change f(x) to y 2 switch x and y ~~3 solve for y~~

1 $y = 2^x$

2 $x = 2^y$

Equation after Step 2: $x = 2^y$

$y = \log_2 x$

To help us with the next step we have to look at a new function called a logarithm.

How to solve for y

1

Defining Logarithmic Equations – The function $y = \log_b x$ is the name we give the inverse of $y = b^x$. For example, $y = \log_2 x$ is the inverse of $y = 2^x$. We can write an **equivalent exponential equation** for each logarithm as follows:

$$y = \log_b x \text{ is the same as } b^y = x$$

or

$$f^{-1}(x) = \log_b x$$

Now let's write the equation on the previous page as a logarithmic equation.

Logarithmic Equation: $f^{-1}(x) = \log_2 x$

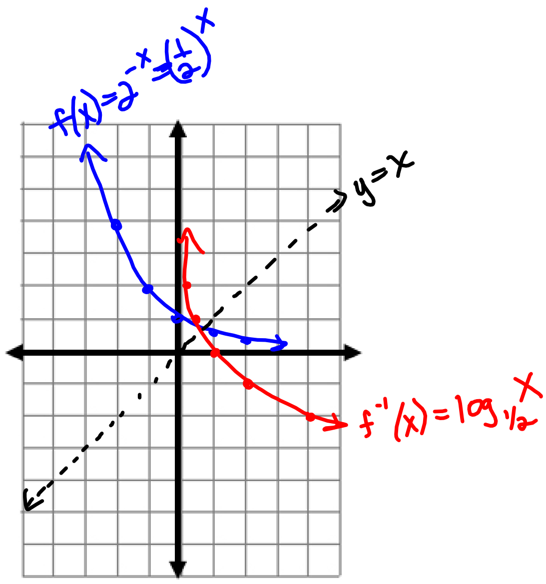
You try:

$f(x) = 2^{-x} = \left(\frac{1}{2}\right)^x$

x	y
-2	4
-1	2
0	1
1	1/2
2	1/4

Inverse: $f^{-1}(x) = \log_{1/2} x$

x	y
4	-2
2	-1
1	0
1/2	1
1/4	2



Logarithmic Equation: $f^{-1}(x) = \log_{1/2} x$

Describe the end behavior of the inv function as $x \to \infty$ $y \to -\infty$

Describe the end behavior of the inv function as $x \to 0$ $y \to \infty$

Summary

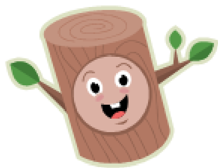
Point on every logarithmic graph: $(1,0)$

Domain: $(0,\infty)$

Quadrants: I, IV

Range: $(-\infty,\infty)$

Asymptote: $x=0$



Pt. Exp: $(0,1)$

D: $(-\infty,\infty)$

Q: I, II

R: $(0,\infty)$

A: $y=0$

Let's graph the function $f(x) = \log_2(x+2)$
(Hint: What type of transformation will occur with this function? Please write your answer below.)

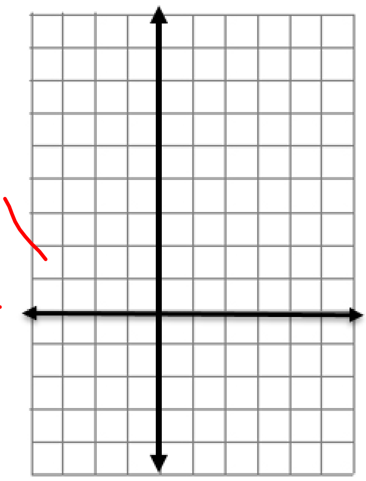
$f(x) = \log_2 x$

x	y

$f(x) = \log_2(x+2)$

x	y

See pg. 1



What transformation happened to the function? left 2

Describe the end behavior of the function as $x \to \infty$

$y \to \infty$

Describe the end behavior of the function as $x \to -2$

$y \to -\infty$

Describe what transformations would occur for the graph $f(x) = \log_3(x-4) + 2$

Rt. 4 and up 2

For the logarithmic function $f(x) = \log_3(x-4) + 2$, explain why $x = 0$ is not in its domain.

$f(0) = \cancel{\log_3(-4)} + 2$

can't take log of negative #.

Logarithmic Form of an Equation

General Rule: $\log_b c = a \leftrightarrow b^a = c$

log form Exp. form

Restrictions b: $b>0$

c: $c>0$

Write in Exponential Form:

1. $\log_2 4 = 2$ $2^2 = 4$
2. $\log_5 125 = 3$ $5^3 = 125$
3. $\log_{10} 100 = x$ $10^x = 100$ $x=2$
4. $\log_{15} 1 = 0$ $15^0 = 1$

Write in Log Form:

5. $3^2 = 9$ $\log_3 9 = 2$

6. $10^{-1} = .1$ $\log_{10} .1 = -1$

7. $4^x = 16$ $\log_4 16 = x$

8. $12^0 = 1$ $\log_{12} 1 = 0$