

## Unit 10 HW 9

$$\textcircled{1} \{.9730\}$$

$$\textcircled{2} \{.2197\}$$

$$\textcircled{3} t = 31.4 \text{ years}$$

$$\textcircled{4} \text{(a)} k = .0193$$

$$\text{(b)} t = 10.4 \text{ years}$$

$$\textcircled{5} \text{(a)} \$1001.86$$

$$\text{(b)} 15 \text{ years}$$

$$\text{(c)} 7.7\%$$

Name \_\_\_\_\_

Alg 2 CC

Date \_\_\_\_\_

Unit10Day9 HW

Solve each equation using natural logarithms. Round your solution to the nearest ten-thousandth.

1.  $4e^{2x} - 11 = 17$

$$\begin{array}{r} +11 +11 \\ 4e^{2x} = 28 \\ \hline 4 \quad 4 \\ e^{2x} = 7 \end{array}$$

$$\frac{2 \times \ln e = \ln 7}{2} \quad \frac{2 \ln e = \ln 7}{2}$$

$$x = .9730$$

2.  $\frac{3e^{5x}}{3} = \frac{9}{3}$

$$e^{5x} = 3$$

$$\frac{5 \times \ln e = \ln 3}{5} \quad \frac{5 \ln e = \ln 3}{5}$$

$$x = .2197$$

3. How many years, to the nearest tenth of a year, would it take for \$2500 to triple if it is earning a continuous compound interest of 3.5% per year?

$$\frac{7500}{2500} = \frac{2500 e^{.035t}}{2500}$$

$$t = 31.4 \text{ years}$$

$$\frac{\ln 3}{.035} = \frac{.035t \ln e}{.035}$$

4. In 2014, the population of Georgia was 9.36 million people. In 2007 it was 8.18 million.

Use  $N = N_0 e^{kt}$ .

- a. Determine the value of  $k$ , Georgia's relative rate of growth. (to 4 decimal places)

$$\frac{9.36}{8.18} = \frac{8.18 e^{7k}}{8.18} \quad k = .0193$$

$$\frac{\ln\left(\frac{9.36}{8.18}\right)}{7} = \frac{7k \cancel{\ln e}}{7}$$

- b. When will Georgia's population reach 10 million people? (Use the value of  $k$  from part a) Round your answer to the nearest tenth of a year.

$$\frac{10}{8.18} = \frac{8.18 e^{.0193t}}{8.18} \quad t = 10.4 \text{ years}$$

$$\frac{\ln\left(\frac{10}{8.18}\right)}{.0193} = \frac{.0193t \cancel{\ln e}}{.0193}$$

5. Use the formula for continuously compounded interest.

- a. If you deposited \$800 in an account paying 4.5% interest compounded continuously, how much money would be in the account in 5 years?

$$A = Pe^{rt}$$

$$A = 800e^{.045(5)}$$

$$A = \$1001.86$$

- b. How long, to the nearest year, will it take to double your money?

$$2 = e^{.045t}$$

$$\frac{\ln 2}{.045} = \frac{.045t \cancel{\ln e}}{.045}$$

$$t = 15.4 \text{ years}$$

$$15 \text{ years}$$

- c. If you want to double your money in 9 years, what rate would you need? Round your rate to the nearest tenth of a percent.

$$\frac{1600}{800} = \frac{800e^{9r}}{800}$$

$$\frac{\ln 2}{9} = \frac{9r \cancel{\ln e}}{9}$$

$$r = .077$$

$$7.7\%$$

## More Applications of Logarithms

## More Applications of Natural Logarithms

## Unit 10 Day 10

A hot liquid is cooling in a room whose temperature is constant. Its temperature can be modeled using the exponential function shown below. The temperature,  $T$ , is in degrees Fahrenheit and is a function of the number of minutes,  $m$ , it has been cooling.

$$T(m) = 101e^{-0.03m} + 67$$

- (a) What was the initial temperature of the water at  $m = 0$ . Do without using your calculator.

$$\begin{aligned} T(0) &= 101e^{-0.03(0)} + 67 \\ &= 101 + 67 = 168^\circ\text{F} \end{aligned}$$

- (c) Using the natural logarithm, determine algebraically when the temperature of the liquid will reach  $100^\circ\text{F}$ . Show the steps in your solution. Round to the nearest tenth of a minute.

$$\begin{aligned} 100 &= 101e^{-0.03m} + 67 \\ -67 &\quad -67 \\ \hline 33 &= 101e^{-0.03m} \\ \frac{33}{101} &= \frac{101e^{-0.03m}}{101} \\ \ln\left(\frac{33}{101}\right) &= \ln e^{-0.03m} \\ \ln\left(\frac{33}{101}\right) &= \frac{-0.03m \ln e}{-0.03} \\ m &= 37.3 \text{ minutes} \end{aligned}$$

- (b) How do you interpret the statement that  $T(60) = 83.7$ ?

At 60 minutes, the temperature of the liquid is  $83.7^\circ\text{F}$ .

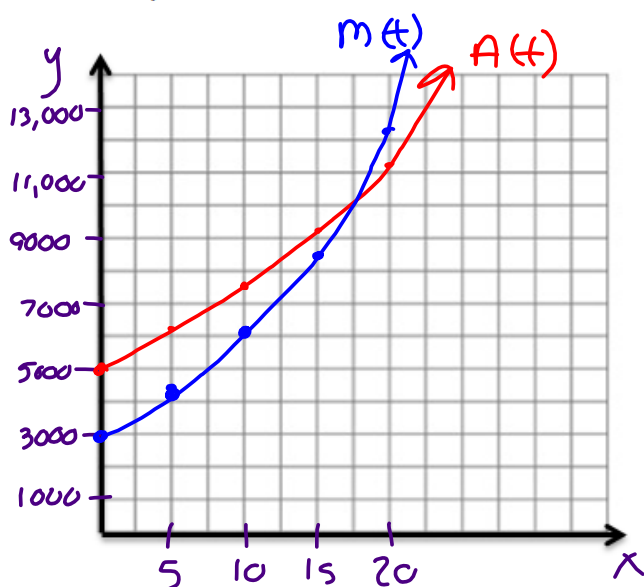
- (d) On average, how many degrees are lost per minute over the interval  $10 \leq m \leq 30$ ? Round to the nearest tenth of a degree.

$$\begin{aligned} \text{Rate of Change } \frac{\Delta y}{\Delta x} \\ T(30) &= 101e^{-0.03(30)} + 67 = 108.06 \\ T(10) &= 101e^{-0.03(10)} + 67 = 141.82 \\ \frac{T(30) - T(10)}{30 - 10} &= \frac{108.06 - 141.82}{20} \\ &= -1.7 \text{ degrees per minute} \end{aligned}$$

Amy and Mark earned a stipend working at a summer camp. They both decided to invest their earnings. Amy earned \$5000 as the camp director and Mark earned \$3000 as a camp counselor. They both wanted their investment to be compounded continuously so they did their research and invested their money. Amy was able to invest her money at 4% compounded continuously and Mark was able to invest his money at 7% compounded continuously.

$$A = Pe^{rt}$$

1. Write two functions,  $A(t)$  and  $M(t)$ , to represent their investments.
2. Graph each function on the set of axes below.



$$A(t) = 5000e^{.04t}$$

$$M(t) = 3000e^{.07t}$$

x	0	5	10	15	20
A(t)	5000	6107	7459	9111	11125

y	0	5	10	15	20
M(t)	3000	4257	6041	8573	12166

3. To the nearest year, when will Mark's investment start to exceed Amy's investment.

Using the graph, use 2<sup>nd</sup> → Trace → Intersect

17 years

(17.027, 9880)

4. How long will it take, to the nearest tenth of a year, for the both of them to double their investments? Only algebraic solutions are acceptable.

$$A(t) = 5000e^{.04t}$$

$$10000 = 5000e^{.04t}$$

$$2 = e^{.04t}$$

$$\frac{\ln 2}{.04} = \frac{.04t}{.04} \quad \text{line}$$

17.3 years

$$M(t) = 3000e^{.07t}$$

$$6000 = 3000e^{.07t}$$

$$2 = e^{.07t}$$

$$\frac{\ln 2}{.07} = \frac{.07t}{.07} \quad \text{line}$$

9.9 years



