Sequences and Series

Arithmetic

$a_n = a_1 + d(n-1)$

Geometric

$a_n = a_1 r^{n-1}$

Geometric Series

$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$

Introduction to Sequences & Arithmetic Sequences **Day 1**

A Sequence is an ordered set of numbers or other items. Each number in the sequence is called a term. For example, in the sequence 4, 7, 10, 13, 16, …, the second term is 7. A sequence can be infinite (without end) or finite (limited number of terms). Each term in the sequence can be paired with a position number, and these pairings establish a function whose domain is the set of position numbers and whose range is the set of terms, as shown below. The position numbers are consecutive integers that typically start at either 0 or 1.

Position number	n	1	2	3	4	5
Term of sequence	f(n)	4	7	10	13	16

For this sequence, we write f(4) = 13, which can be interpreted as "the fourth term of the sequence is 13."

4, 7, 10, 13, 16 … Can you find a pattern?

+3

In this sequence, each term is 3 more than the previous term.

This is an arithmetic sequence because successive terms differ by the **same** number, called the common difference (d and d ≠ 0).

3 is called the common difference (d) $d = a_2 - a_1$

4 is called the 1st term (a_1)

7 is called the 2nd term (a_2)

16 is called the 5th term (a_5)

Now, we'll find a formula (rule) for the n^{th} term (a_n).

1st term:	$a_1 =$ $4+3(0)$	$= 4$
2nd term:	$a_2 =$ $4+3(1)$	$= 7$
3rd term:	$a_3 =$ $4+3(2)$	$= 10$
4th term:	$a_4 =$ $4+3(3)$	$= 13$
5th term:	$a_5 =$ $4+3(4)$	$= 16$
n^{th} term:	$a_n =$ $4+3(n-1)$ or $4+3n-3$ $3n+1$	

n^{th} term, 1st term, common diff

Arithmetic Rule:

$$a_n = a_1 + d(n-1)$$

In Function Notation:

$$f(n) = f(1) + d(n-1)$$

1. Is the sequence arithmetic? If so, find d, an n^{th} term formula (a_n) and the next three terms.

a. 1.9, 1.2, 0.5, -0.2, -0.9...

YES

$d = -.7$ $\quad d = a_2 - a_1 = 1.2 - 1.9 = -.7$

$a_n = a_1 + d(n-1)$

$a_n = 1.9 - .7(n-1)$

$a_6 = 1.9 - .7(6-1) = -1.6$

$a_7 = 1.9 - .7(7-1) = -2.3$

$a_8 = 1.9 - .7(8-1) = -3$

b. 3/4, 3/8, 3/16, 3/32, 3/64 ...

NO

Not arithmetic but it is a sequence

The previous examples used an Explicit Formula which defines the nth term of a sequence as a function of n. Sequences can also be described by using a Recursive Formula which defines the nth term of a sequence as a function of one or more previous terms. On the previous page, we wrote the explicit formula $f(n) = 4 + 3(n-1)$ for the arithmetic sequence 4, 7, 10, 13, 16 ... $a_n = 4+3(n-1)$

Use the following recursive formula to find the first 4 terms of the same arithmetic sequence.

nth term, term before it

$f(n) = f(n-1) + 3$ with $f(1) = 4$

$f(1) = 4$

$f(2) = f(1) + 3$
$= 4 + 3$
$= 7$

$f(3) = f(2) + 3$
$= 7 + 3$
$= 10$

$f(4) = f(3) + 3$
$= 10 + 3$
$= 13$

2. Find an explicit and recursive formula rule for the n^{th} term (a_n). Then find the next term with your recursive formula.

a. 9.2, 9.15, 9.1, 9.05, 9... $d = -.05$

E: $a_n = a_1 + d(n-1)$

$a_n = 9.2 - .05(n-1)$

R: $\begin{cases} a_n = a_{n-1} - .05 \\ a_1 = 9.2 \end{cases}$ (my term; the term before)

$a_6 = a_5 - .05$

$a_6 = 9 - .05$

$a_6 = 8.95$

b. 4/3, 5/3, 2, 7/3, 8/3... $d = \frac{1}{3}$

E: $a_n = a_1 + d(n-1)$

$a_n = \frac{4}{3} + \frac{1}{3}(n-1)$

R: $\begin{cases} a_n = a_{n-1} + \frac{1}{3} \\ a_1 = \frac{4}{3} \end{cases}$

$a_6 = a_5 + \frac{1}{3}$

$a_6 = \frac{8}{3} + \frac{1}{3} = \frac{9}{3} = 3$

3. Find the 12th term of the arithmetic sequence by using a formula.

a. 32, 25, 18, 11, 4... $d = -7$

$a_n = a_1 + d(n-1)$

$a_n = 32 - 7(n-1)$

$a_{12} = 32 - 7(12-1)$

$a_{12} = 32 - 77$

$a_{12} = -45$

b. -9, -7, -5, -3, -1... $d = 2$

$a_n = a_1 + d(n-1)$

$a_n = -9 + 2(n-1)$

$a_{12} = -9 + 2(12-1) = -9 + 22 = 13$

4. Find the 6th term of the arithmetic sequence using the given terms and a formula. ~~Do b first.~~

a. $a_8 = -4$ and $a_{11} = 14$

$d = \frac{a_{11} - a_8}{11 - 8}$

(*) $a_{11} = a_8 + 3d$

$14 = -4 + 3d$

$18 = 3d$

$6 = d$

$a_8 = a_6 + 2d$

$-4 = a_6 + 2(6)$

$-4 = a_6 + 12$

$-16 = a_6$

or

$a_6 = a_8 - 2d$

b. $a_3 = 20.5$ and $a_8 = 13$

$a_8 = a_3 + 5d$

$13 = 20.5 + 5d$

$-7.5 = 5d$

$-1.5 = d$

$a_6 = a_3 + 3d$

$a_6 = 20.5 + 3(-1.5)$

$a_6 = 16$