HW 11-3

1) 56 2) 18 3) -3

4) 15.5 5) -3 6) 6

7) $\frac{163}{60}$ 8) $\frac{25}{16}$ 9) 278

10) $\sum_{n=1}^{7} -2(-2)^{\rightarrow n-1}$ 11) $\sum_{k=1}^{10} \frac{1}{k^2}$

12) $\sum_{k=1}^{10} 5k - 1$ 13) (3)

14) 160

15a) $a_n = x^{n-1}(x + 1)$ b) {0,-1}

Name Key [redacted] HW 11-3

For 1 - 9, expand and evaluate.

1. $\sum_{i=2}^{5} 4i$

$= 4(2)+4(3)+4(4)+4(5)$
$= 8+12+16+20$
$= 20+36$
$= 56$

2. $\sum_{k=0}^{3}(k^2+1)$

$0^2+1=1$
$1^2+1=2$
$2^2+1=5$
$3^2+1=10$
18

3. $\sum_{j=-2}^{0}(2j+1)$

$2(-2)+1=-3$
$2(-1)+1=-1$
$2(0)+1=1$
-3

4. $\sum_{i=-1}^{3} 2^i$

$= 2^{-1}+2^0+2^1+2^2+2^3$
$= \frac{1}{2}+1+2+4+8$
$= 15.5$

5. $\sum_{i=0}^{2}(-1)^{2i+1}$

$(-1)^{2(0)+1}=-1$
$(-1)^{2(1)+1}=-1$
$(-1)^{2(2)+1}=-1$
-3

6. $\sum_{i=1}^{3}\log(10^i)$

$= \log 10^1+\log 10^2+\log 10^3$
$= 1+2+3 = 6$

7. $\sum_{n=1}^{4}\frac{n}{n+1}$

$= \frac{1}{1+1}+\frac{2}{2+1}+\frac{3}{3+1}+\frac{4}{4+1}$
$= \frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}$ lcd=60
$= \frac{30}{60}+\frac{40}{60}+\frac{45}{60}+\frac{48}{60} = \frac{163}{60}$

8. $\dfrac{\sum_{i=2}^{4}(i+1)^2}{\sum_{i=2}^{4}(i^2+1)}$

num $= (2+1)^2+(3+1)^2+(4+1)^2$
$= 3^2+4^2+5^2 = 50$
$= 9+16+25 = 50$
den $= (2^2+1)+(3^2+1)+(4^2+1)$
$= 5+10+17 = 32$
$50/32 = 25/16$

9. $\sum_{n=0}^{3} 256^{\frac{1}{2^n}}$

$256^{\frac{1}{2^0}} = 256$
$256^{\frac{1}{2^1}} = \sqrt{256} = 16$
$256^{\frac{1}{2^2}} = \sqrt[4]{256} = 4$
$256^{\frac{1}{2^3}} = \sqrt[8]{256} = 2$
278

For 10 - 12, write each of the sums using summation/sigma notation. Use k as your index variable. Remember there are many correct ways to write each sum. $+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}+\frac{1}{7^2}+\frac{1}{8^2}+$

10) -2 + 4 + -8 + ... + 64 + -128

$+16+-32+$

Geom, $r=-2$

$a_n = a_1 r^{n-1}$

G $a_n = -2(-2)^{n-1}$

$$\boxed{\sum_{n=1}^{7} -2(-2)^{n-1}}$$

11) k= 1 2 3 10

$\frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\ldots+\frac{1}{81}+\frac{1}{100}$

$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2} \quad \frac{1}{9^2}+\frac{1}{10^2}$

$\frac{1}{k^2}$

$$\boxed{\sum_{k=1}^{10}\frac{1}{k^2}}$$

12) 4 + 9 + 14 + ... + 44 + 49 Arith, $d=5$

$+19+24+29+34+39+$

A $a_n = a_1 + d(n-1)$

$a_n = 4+5(n-1) = 4+5n-5$

$a_n = 5n-1$. Use k →

$$\boxed{\sum_{k=1}^{10}(5k-1)}$$

$4+5(n-1)$

13. Which of the following represents the sum 2 + 5 + 10 + ...+ 82 + 101 ?

~~(1)~~ $\sum_{j=1}^{6}(4j-3)$ $= 1 +$

(3) $\sum_{j=1}^{10}(j^2+1)$ $2+5+10+$

~~(2)~~ $\sum_{j=1}^{101}(j-2)$ $= 1 + 2 + 3$

~~(4)~~ $\sum_{j=0}^{11}(4^j+1)$ $2+5+17$

14. A sequence is defined recursively by: $b_n = 4b_{n-1} - 2b_{n-2}$ with $b_1 = 1$ and $b_2 = 3$. What is the value of $\sum_{j=3}^{5} b_j$? Justify your answer by showing all work.

$= b_3 + b_4 + b_5$

$b_3 = 4b_2 + 2b_1 = 4(3) - 2(1) = 10$

$b_4 = 4b_3 + 2b_2 = 4(10) - 2(3) = 34$

$b_5 = 4b_4 + 2b_3 = 4(34) - 2(10) = 116$

160

From Spring 2015 Sample Questions # 11

15a) Write an explicit formula for a_n, the nth term of the recursively defined sequence below. (tip – write out at least the first 3 terms)

$a_1 = x + 1$
$a_n = x(a_{n-1})$

$a_1 = x + 1$
$a_2 = x(x+1)$
$a_3 = x(x(x+1))$

Geom, $r = x$

$a_n = a_1(r)^{n-1}$
$a_n = (x+1)(x)^{n-1}$
or $a_n = x^{n-1}(x+1)$

b. For what values of x would $a_n = 0$ when $n > 1$? So $n \neq 1$

when is $a_2 = 0$, $x = ?$

$a_2 = x(x+1)$
$x(x+1) = 0$
$x = 0 \mid x = -1$

$a_3 = x^2(x+1)$
$x^2(x+1) = 0$
$x = 0 \mid x = -1$

this continues

$\therefore x = \{0, -1\}$

Geometric Series **Day 4**

Warm-up:

The sum of a geometric sequence is called a Geometric Series.

Given a geometric series defined by the recursive formula $a_1 = 3$ and $a_n = a_{n-1} \bullet 2$, determine the value of

$S_5 = \sum_{i=1}^{5} a_i = a_1 + a_2 + a_3 + a_4 + a_5$

↑

Sum of the first 5 terms

$S_5 = 3+6+12+24+48 = 93$

$a_1 = 3$

$a_2 = 3 \cdot 2 = 6$

$a_3 = 6 \cdot 2 = 12$

$a_4 = 12 \cdot 2 = 24$

$a_5 = 24 \cdot 2 = 48$

1. Let's derive a formula for finding this sum.

Recall for a geometric sequence, the nth term formula is $a_n = a_1 \bullet r^{n-1}$. So, the general form of a geometric series is: $S_n = a_1 + \cancel{a_1 r} + \cancel{a_1 r^2} + \ldots + \cancel{a_1 r^{n-2}} + \cancel{a_1 r^{n-1}}$

(a) Write an expression below for the product of r and S_n.

$r \bullet S_n = \cancel{ra_1} + \cancel{a_1 r^2} + \cancel{a_1 r^3} + \cdots + \cancel{a_1 r^{n-1}} + a_1 r^n$

(b) Find, in simplest form, the value of $S_n - r \bullet S_n$ in terms of a_1, r, and n.

$S_n - r \bullet S_n = a_1 - a_1 r^n$

(all other terms cancel)

(c) Write both sides of the equation in (b) in their factored form.

$S_n - rS_n = a_1 - a_1 r^n$

$S_n(1-r) = a_1(1-r^n)$ $\longrightarrow$

(d) From the equation in part (c), find a formula for S_n in terms of a_1, r, and n.

$$\boxed{S_n = \frac{a_1(1-r^n)}{1-r}}$$

2. Use this formula to check your answer to the warm-up above.

$a_1 = 3 \quad r = 2 \quad n = 5$

$S_5 = \frac{3(1-2^5)}{\cancel{1-2}\ -1} = -3(1-2^5) = 93$

SUM OF A FINITE GEOMETRIC SERIES

For a geometric series defined by its first term, a_1, and its common ratio, r, the sum of n terms is:

$$S_n = \frac{a_1(1-r^n)}{1-r} \quad \text{or} \quad \frac{a_1 - a_1 r^n}{1-r}$$

$r > 1 \rightarrow$ growth

$r < 1 \rightarrow$ decay

3. Determine the sum of a geometric series with 8 terms whose first term is 3 and whose common ratio is 4.

S_8 $a_1=3$ $r=4$ $n=8$

$$S_n=\frac{a_1(1-r^n)}{1-r}=\frac{3(1-4^8)}{1-4}=65,535$$

4. Find the value of the geometric series shown below. Show all work.

Change:

$a_1=-6$

$r=-2$

$n=8$

48 -96 192 -384

$-6+12-24+\ldots+768$ Not +

$$S_8=\frac{-6(1-(-2)^8)}{1-(-2)}$$

$S_8=510$

NORMAL FLOAT AUTO REAL RADIAN MP

$^{-}3(1-2^5)$

93

$\frac{3(1-4^8)}{1-4}$

65535

$\frac{^{-}6(1-(^{-}2)^8)}{1-^{-}2}$

510

1, 2, 4 .01, .02, .04

5. A person places 1 penny in a piggy bank on the first day of the month, 2 pennies on the second day, 4 pennies on the third, and so on. Will this person be a millionaire at the end of a 31 day month? Show all work.

$n = 31 \quad a_1 = .01 \quad r = 2$

$$S_{31} = \frac{.01(1 - 2^{31})}{1 - 2}$$

$$= \$21{,}474{,}836.47$$

Yes

Not realistic!!!!