HW 11-4

1) 19,680 2) 39.375 3) 1,705

4) (1) 5) (4) 6) 19,171

7) 6

8) $\frac{29,524}{729}$

9) $10,737,418.23

10) 2,778

11) 63

Name Key HW 11-4 Alg 2 HW 9-4

For 1 - 3, find the sums of geometric series with the following properties by using the formula. $S_n = \frac{a_1(1-r^n)}{1-r}$

1. $a_1 = 6, r = 3, n = 8$

$S_8 = \frac{6(1-3^8)}{1-3}$

$S_8 = 19,680$

2. $a_1 = 20, r = \frac{1}{2}, n = 6$

$S_6 = \frac{20(1-.5^6)}{1-.5}$

$S_6 = 39.375$

3. $a_1 = -5, r = -2, n = 10$

$S_{10} = \frac{-5(1-(-2)^{10})}{(1-(-2))}$

$S_{10} = 1705$

4. If the geometric series $54 + 36 + ... + \frac{128}{27}$ has seven terms in its sum, then the value of the sum is

$a_1 =$ $n = 7$, $r = \frac{36}{54} = 2/3$

(1) $\frac{4118}{27}$ (2) $\frac{1274}{3}$ (3) $\frac{1370}{9}$ (4) $\frac{8241}{54}$

$S_7 = \frac{54(1-(2/3)^7)}{1-2/3}$

$S_7 = \frac{4118}{27}$

5. A geometric series has a first term of 32 and a final term of $-\frac{1}{4}$ and a common ratio of $-\frac{1}{2}$. The value of this series is

$r = -1/2$ a_1 (1) Need n.

(1) $-\frac{1}{4} = 32(-\frac{1}{2})^{n-1}$ → $\frac{-1}{128} = (-\frac{1}{2})^{n-1}$

$\frac{-\frac{1}{4}}{32}$ $\quad$ 32

$(-2)^{-7} = (-2^{-1})^{n-1}$

$-7 = -n+1$

$n = 8$

(1) 19.75 (2) 16.25 (3) 22.5 (4) 21.25

(2) $S_8 = \frac{32(1-(-.5)^8)}{(1-(-.5))} = 21.25$

6. Using the geometric series formula, determine the value of $\sum_{k=0}^{8} 256\left(\frac{3}{2}\right)^k$. Carefully consider the number of terms in this series. $a_1 = 256$, $r = 3/2$, $n = 9$

$k = 0, 1, 2, 3, 4, 5, 6, 7, 8$

$S_9 = \frac{256(1-(1.5)^9)}{(1-1.5)} = 19,171$

7. A geometric series whose first term is 3 and whose common ratio is 4 sums to 4095. Find the number of terms in this sum.

a_1 $\quad$ $r = 4$

$4095 = \frac{3(1-4^n)}{1-4}$

$4095 = \frac{\cancel{3}(1-4^n)}{-\cancel{3}-1}$

$\frac{-4095}{-1} = \frac{1-4^n}{-1}$

$-4096 = -4^n$

$4096 = 4^n$

$4^6 = 4^n$

$n = 6$

8. Find the sum of the geometric series below. Show all work to justify your answer.

$27 + 9 + 3 + \ldots + \frac{1}{729}$

① $a_1 = 27$, $r = \frac{9}{27} = \frac{1}{3}$, $n =$ ____

② $729\left(\frac{1}{729}\right) = \left(27\left(\frac{1}{3}\right)^{n-1}\right)729$

$1 = 19683\left(\frac{1}{3}\right)^{n-1}$

$\frac{1}{19683} = (3^{-1})^{n-1}$

$3^{-9} = (3^{-1})^{n-1}$

$-9 = -n + 1$

$n = 10$

$S_{10} = \frac{27(1-(\frac{1}{3})^{10})}{(1-\frac{1}{3})}$

$S_{10} = \frac{29524}{729}$

9. If you are paid a salary of \$.01 on the first day of April and \$.02 on the second day, your salary continues to double each day, how much will you earn in the month of April? Show use of a formula.

Geom. $r = 2$, $a_1 = .01$, April has 30 days, $= n$

$S_{30} = \frac{.01(1-2^{30})}{(1-2)} = \$10,737,418.23$

* sequence not series

10. The population of a town is 2400 in 2007. From 2007 to 2010, the population of the town increased by 5% per year. What's the population of the town in 2010? Round your answer to the nearest whole number. → $r = 1.05$, not a series but a sequence.

2007 → 2400 population

2010 → ____ "

a_1 (2007), a_2 (2008), a_3 (2009), a_4 (2010) → $n = 4$

$a_n = a_1 r^{n-1}$

$a_n = 2400(1.05)^{n-1}$

$a_4 = 2400(1.05)^3 = 2778.3 \approx 2778$

11. There are 64 teams in the NCAA basketball tournament. How many games must be played to select a winner? You must show use of a formula.

Teams	64	32	16	8	4	2
Games	32	16	8	4	2	1

$r = \frac{1}{2}$ → Game sum = 63 Geom.

Formula

$a_n = 32(.5)^{n-1}$

$a_1 = 32$, $r = .5$, $n = 6$

$S_6 = \frac{32(1-(.5)^6)}{(1-.5)} = 63$

Geometric Modeling Applications **Day 5**

Warm-up:

1. In a geometric sequence, the first term is -2 and the common ratio is 3. Find the sum of the first 10 terms.

$a_1 = -2$
$r = 3$
$n = 10$

$$S_{10} = \frac{\cancel{-2}(1-3^{10})}{\cancel{1-3}} = 1-3^{10} = -59{,}048$$

2. Find the sum of the geometric series: $9+3+1+\frac{1}{3}+\frac{1}{9}+\ldots+\frac{1}{27}$

$n=6$
$a_1 = 9$
$r = \frac{1}{3}$

(1) $8\frac{13}{27}$ (2) $8\frac{25}{27}$ (3) $13\frac{13}{27}$ (4) $13\frac{25}{27}$

$$S_6 = \frac{9(1-(\frac{1}{3})^6)}{1-\frac{1}{3}} = \frac{364}{27}$$

Modeling Examples

1. A culture of bacteria is growing at a rate of 8% per day. If there are 150 bacteria ~~in the initial~~ at the end of day 1 population, approximately how many bacteria, to the nearest bacteria, will there be in 7 weeks?

$r = 1.08$

$a_1 = 150$

$n = 7 \times 7 = 49$ days

$a_n = a_1 r^{n-1}$

$a_{49} = 150(1.08)^{49-1} = 6031.586$

6032 bacteria

2. Devin began running a month ago to get back in shape. The first day he ran .5 miles. Each day after that, he ran 10% more than the previous day for a total of 30 days. Use the formula for the sum of a finite geometric series to calculate the total distance Devin ran over the 30 days. Round to the nearest thousandth of a mile.

$a_1 = .5$
$r = 1.10$
$n = 30$

$$S_{30} = \frac{.5(1 - 1.1^{30})}{1 - 1.1} = 82.247 \text{ mi}$$

3. Taylor earns \$35,000 in her first year of teaching and earns 4% increase in each successive year. Write a geometric series formula, S_n, for Taylor's total earnings over n years. Then find Taylor's total earnings to the nearest dollar at the end of 10 years of teaching.

$a_1 = 35{,}000$
$r = 1.04$

① $S_n = \dfrac{35{,}000(1-1.04^n)}{1-1.04}$

② $S_{10} = \dfrac{35{,}000(1-1.04^{10})}{1-1.04}$

$= \$420{,}214$

4. Monthly mortgage payments can be calculated according to the formula, $A = \frac{Mp^{nt}(1-p)}{(1-p^{nt})}$ where M is the size of the mortgage, n is the number of compounds per year, t is the length of the mortgage, in years, and $p = \left(1 + \frac{r}{n}\right)$ where r is the interest rate as a decimal. What would the monthly mortgage payments be on $175,000 (M), 15 (t) year mortgage with 6% ($r = .06$) interest, compounded monthly ($n = 12$), to the nearest dollar.

$P = 1 + \frac{.06}{12} = 1.005 \rightarrow X$

$A = \frac{175{,}000X^{12(15)}(1-X)}{1-X^{12(15)}}$

$A = \$\cancel{1476.7}$ \$1477

2) Want a payment of \$1200. How much borrow?

$1200 = \frac{M\boxed{X^{12(15)}(1-X)}}{\boxed{1-X^{12(15)}}}$

$\frac{1200}{.0084\ldots} = \frac{M(.0084\ldots)}{\cancel{.0084\ldots}}$

$M = \$142{,}204$