HW 11-5

1) 36,172

2) 23.75, $T_n = 2\left(\frac{8(1-.5^n)}{(1-.5)}\right) - 8$ where n=7

or $2\left(\frac{4(1-.5)^6}{1-.5}\right)+8$

3) 1,000,000

4) (1)

5) $1729

$\downarrow 8 \uparrow 4 \downarrow 4 \uparrow 2 \downarrow 2 \uparrow 1 \downarrow 1 \uparrow .5 \downarrow \uparrow .25 \downarrow .25 \uparrow \downarrow .125$

Series

1. A new website got 4000 page views on the first day. During the next 4 days, the number of page views increased by 30% per day. What's the total amount of page views in the first 5 days? Round your answer to the nearest whole number.

$a_1 = 4000$, $r = 1 + .30 = 1.30$, $n = 5$

$S_n = \frac{a_1(1-r^n)}{(1-r)}$ $S_5 = \frac{4000(1-1.3^5)}{(1-1.3)} = 36{,}172.4$

$\approx 36{,}172$

Series

2. A ball is dropped from a height of 8 feet. On each bounce it rises to half its previous height. When the ball hits the ground for the seventh time, how far has it traveled (including going up and down)? Write terms out to help. Create a formula.

$\downarrow 8, \uparrow 4, \downarrow 4, \uparrow 2, \downarrow 2, \uparrow 1, \downarrow 1, \uparrow .5, \downarrow .5, \uparrow .25, \downarrow .25, \uparrow .125, \downarrow .125$

(1) (2) (3) (4) (5) (6) (7)

$\boxed{2(S_n) - 8} = 2\left(\frac{8(1-.5^7)}{(1-.5)}\right) - 8 = 2(15.875) - 8 = 23.75$

$a_1 = 8$, $r = .5$, $n = 7$

formula

$T_n = 2\left(\frac{8(1-.5^n)}{(1-.5)}\right) - 8$ where $n = 7$

sequence

3. Gill Bate's personal fortune doubles every year. If the value of his fortune was estimated at $32,000,000 in 2000, how much was it in 1995.

① $a_1(1995), a_2(1996), a_3(1997), a_4(1998), a_5(1999), a_6(2000)$

$n = 6$

② $a_n = a_1(2)^{n-1}$

$a_6 = a_1(2)^{6-1}$

$32{,}000{,}000 = a_1(2)^5$

$a_1 = \frac{32{,}000{,}000}{2^5 \; 32} =$ \$1,000,000 in 1995

Sum or series $S_n = \frac{a_1(1-r^n)}{(1-r)}$

4. Kyle wants to increase his running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kyle runs 6 miles in week one, which expression can help him find the total number of miles he will have run over the course of his 8-week training program?

$r = 1.10 \quad n = 8$

$a_1 = 6$

(1) $\sum_{n=1}^{8} 6(1.10)^{n-1}$

(3) $\frac{6-6(1.10)^8}{\cancel{0.90}}$

$S_8 = \frac{6(1-1.10^8)}{(1-1.10)}$

(2) $\sum_{n=1}^{8} 6(1.10)^n$

(4) $\frac{6-6(1.10)^n}{\cancel{1.10}}$

$S_8 = \frac{6-6(1.10)^8}{-.10}$

not a choice

$a_n = a_1(r)^{n-1}$

$a_n = 6(1.10)^{n-1}$ 8 weeks

$* \sum_{n=1}^{8} 6(1.10)^{n-1} \rightarrow (1)$

5. Monthly mortgage payments can be calculated according to the formula, $A = \frac{Mp^{nt}(1-p)}{(1-p^{nt})}$ where M is the size of the mortgage, n is the number of compounds per year, t is the length of the mortgage, in years, and $p = \left(1+\frac{r}{n}\right)$ where r is the interest rate as a decimal and p is rounded to the nearest thousandth. Determine the monthly mortgage payments on \$190,000, 15 year mortgage with 7% interest, compounded monthly, to the nearest dollar.

$r = .07 \quad n = 12 \quad m \quad 12 \times 15 = 180$

$1.006 \rightarrow x$

① $p = 1 + \frac{.07}{12} = 1.0058 \approx 1.006$

② $A = \frac{190{,}000(1.006)^{(180)}(1-1.006)}{(1-1.006^{180})}$

$A = 1729.08 \approx \$1729$

Unit 11 Day 6 Key

1. (2)
2. (4)
3. 171.958 miles
4. (4)
5. 3, 7, 15, 31
   No. There's no common ratio between terms.
6. (2)
7. $624
8. (3)
9. (4)
10. 639
11. $a_n = 1 + .25n$ or equivalent;
    $16.00
12. (3)
13. $1,247; $20,407

Sequence & Series Regents Practice **Unit 11 Day 6**

1. (January 2019)
When a ball bounces, the heights of consecutive bounces form a geometric sequence. The height of the first bounce is 121 centimeters and the height of the third bounce is 64 centimeters. To the nearest centimeter, what is the height of the fifth bounce?

(1) 25 (3) 36

(2) 34 (4) 42

a_1 121 a_2 a_3 64

$a_n = a_1 \cdot r^{n-1}$

$a_3 = a_1 \cdot r^{3-1}$

$64 = 121 \cdot r^2$

$\pm\sqrt{\frac{64}{121}} = \sqrt{r^2}$

$r = \pm 8/11$

$a_5 = 121(8/11)^4$

$a_5 = 33.851 \approx 34$

2. (January 2019)
Savannah just got contact lenses. Her doctor said she can wear them 2 hours the first day, and can then increase the length of time by 30 minutes each day. If this pattern continues, which formula would not be appropriate to determine the length of time, in either minutes or hours, she could wear her contact lenses on the nth day?

$a_1 = 2$ hrs, $d = .5$ hr

$a_1 = 120$ mins, $d = 30$ mins

(1) $a_1 = 120$
$a_n = a_{n-1} + 30$

Recursive mins yes ✓

(3) $a_1 = 2$
$a_n = a_{n-1} + 0.5$

Recursive hrs. yes ✓

(2) $a_n = 90 + 30n$

Explicit mins yes ✓

$a_n = 120 + 30(n-1)$

$a_n = 120 + 30n - 30$

$a_n = 90 + 3n$

(4) $a_n = 2.5 + 0.5n$

Explicit hrs NO

$a_n = 2 + .5(n-1)$

$a_n = 2 + .5n - .5$

$a_n = 1.5 + .5n$

3. (January 2019)
Rowan is training to run in a race. He runs 15 miles in the first week, and each week following, he runs 3% more than the week before. Using a geometric series formula, find the total number of miles Rowan runs over the first ten weeks of training, rounded to the nearest thousandth. 3 decimals

$a_1 = 15$ miles, $r = 1.03$

$$S_n = \frac{a_1 - a_1 r^n}{1 - r} \qquad S_{10} = \frac{15 - 15(1.03)^{10}}{1 - 1.03} = 171.95818$$

≈ 171.958 miles

4. (August 2018)
The average depreciation rate of a new boat is approximately 8% per year. If a new boat is purchased at a price of \$75,000, which model is a recursive formula representing the value of the boat n years after it was purchased?

down 8%, .08
remain 92%, .92

Explicit
~~(1)~~ $a_n = 75{,}000(0.08)^n$

Explicit
~~(3)~~ $a_n = 75{,}000(1.08)^n$

~~(2)~~ $a_0 = 75{,}000$
$a_n = (0.92)^n$

(4) $a_0 = 75{,}000$
$a_n = 0.92(a_{n-1})$

previous term

5. (June 2018)
The recursive formula to describe a sequence is shown below.

$$a_1 = 3$$
$$a_n = 1 + 2a_{n-1}$$

State the first four terms of this sequence. $a_1 = 3$

$a_2 = 1+2(3) = 7$

$a_3 = 1+2(7) = 15$

$a_4 = 1+2(15) = 31$

$\boxed{3, 7, 15, 31}$

Can this sequence be represented using an explicit geometric formula? Justify your answer.

$\frac{7}{3} \neq \frac{15}{7}$

$2.3 \neq 2.1428$

No. An explicit geometric formula cannot be represented because there is no common ratio between the terms.

6. (January 2018)

Brian deposited 1 cent into an empty non-interest bearing bank account on the first day of the month. He then additionally deposited 3 cents on the second day, 9 cents on the third day, and 27 cents on the fourth day. What would be the total amount of money in the account at the end of the 20th day if the pattern continued?

1, 3, 9, 27

$r = 3$

Geometric

Sum

(1) \$11,622,614.67 $= a_{20}$!

(2) \$17,433,922.00

(3) \$116,226,146.80

(4) \$1,743,392,200.00

$$S_n = \frac{a_1 - a_1 r^n}{1-r} \qquad S_{20} = \frac{1 - 1(3)^{20}}{1-3} = 1743392200 \text{ cents}$$

$= \$17,433,922.00$

7. (June 2018)

The Wells family is looking to purchase a home in a suburb of Rochester with a 30-year mortgage that has an annual interest rate of 3.6%. The house the family wants to purchase is \$152,500 and they will make a \$15,250 down payment and borrow the remainder. Use the formula below to determine their monthly payment, to the *nearest dollar.*

$$M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^n}{\left(1+\frac{r}{12}\right)^n - 1}$$

Find

M = monthly payment
P = amount borrowed
r = annual interest rate
n = total number of monthly payments

$P = 152{,}500 - 15{,}250$
$P = 137{,}250$
$r = .036$
$n = 30\text{yrs} \cdot 12 \text{ months}$
$n = 360$ months or payments

$$M = \frac{137{,}250\left(\frac{.036}{12}\right)\left(1+\frac{.036}{12}\right)^{360}}{\left(1+\frac{.036}{12}\right)^{360} - 1} = \$624$$

8. (January 2018)

At her job, Pat earns \$25,000 the first year and receives a raise of \$1000 each year. The explicit formula for the nth term of this sequence is $a_n = 25{,}000 + (n - 1)1000$. Which rule best represents the equivalent recursive formula?

Arithmetic

$a_1 = 25{,}000$

$d = 1{,}000$

(1) ~~$a_n = 24{,}000 + 1000n$~~

(2) ~~$a_n = 25{,}000 + 1000n$~~

(3) $a_1 = 25{,}000,\ a_n = a_{n-1} + 1000$

(4) $a_1 = 25{,}000,\ a_n = a_{n+1} + 1000$

both explicit

not the previous term

9. (August 2017)

A ball is dropped from a height of 32 feet. It bounces and rebounds 80% of the height from which it was falling. What is the total downward distance, in feet, the ball traveled up to the 12th bounce?

32 B_1 B_2

$r = .8$

$a_1 = 32$

Sum!

(1) 29

(2) 58

(3) 120

(4) 149

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

$$S_{12} = \frac{32 - 32(.8)^{12}}{1 - .8} = 149.004 \approx 149$$

10. (August 2017)
While experimenting with her calculator, Candy creates the sequence 4, 9, 19, 39, 79, … .
Write a recursive formula for Candy's sequence.

$a_1=4, \quad a_n=2a_{n-1}+1$

$$\begin{cases} a_1=4 \\ a_2=9=2(4)+1 \\ a_3=19=2(9)+1 \\ a_4=39=2(19)+1 \\ a_5=79=2(39)+1 \end{cases}$$

Determine the eighth term in Candy's sequence.

$a_5=79$
$a_6=2(79)+1=159$
$a_7=2(159)+1=319$
$a_8=2(319)+1=639$

11. (August 2017)
Simon lost his library card and has an overdue library book. When the book was 5 days late, he owed \$2.25 to replace his library card and pay the fine for the overdue book. When the book was 21 days late, he owed \$6.25 to replace his library card and pay the fine for the overdue book. Suppose the total amount Simon owes when the book is n days late can be determined by an arithmetic sequence. Determine a formula for a_n, the nth term of this sequence.

$a_5 = 2.25$

$a_{21} = 6.25$

$a_n = a_1 + d(n-1)$

Need d

$d = \dfrac{a_{21} - a_5}{21 - 5}$

$d = \dfrac{6.25 - 2.25}{16} = \dfrac{4}{16} = .25$

Need a_1

$a_5 = a_1 + d(5-1)$

$2.25 = a_1 + .25(4)$

$a_1 = 2.25 - 1$

$a_1 = 1.25$

Formula

$a_n = 1.25 + .25(n-1)$

$a_n = 1.25 + .25n - .25$

$a_n = 1 + .25n$

Use the formula to determine the amount of money, in dollars, Simon needs to pay when the book is 60 days late.

$n = 60$

$a_{60} = 1 + .25(60) = \$16.00$

12. (June 2017)

Given $f(9) = -2$, which function can be used to generate the sequence $-8, -7.25, -6.5, -5.75, \ldots$?

(1) $f(n) = -8 + 0.75n$

(2) $f(n) = -8 - 0.75(n - 1)$

(3) $f(n) = -8.75 + 0.75n$

(4) $f(n) = -0.75 + 8(n - 1)$

Arithmetic

$d = a_2 - a_1 = -7.25 - {}^-8 = .75$

$f(n) = a_1 + d(n-1)$

$f(n) = -8 + .75(n-1)$

$f(n) = -8 + .75n - .75$

$f(n) = -8.75 + .75n$

13. (June 2017)

Jim is looking to buy a vacation home for $172,600 near his favorite southern beach. The formula to compute a mortgage payment, M, is $\boxed{M = P \bullet \frac{r(1+r)^N}{(1+r)^N - 1}}$ where P is the principal amount of the loan, r is the monthly interest rate, and N is the number of monthly payments. Jim's bank offers a monthly interest rate of 0.305% for a 15-year mortgage.

$r = .00305$

$P = 172,600$

$N = 15 \times 12 = 180$

With no down payment, determine Jim's mortgage payment, rounded to the *nearest dollar*.

$$M = \frac{172,600 \cdot (.00305(1+.00305)^{180})}{(1+.00305)^{180} - 1}$$

$$M = 1247.49$$

$$M \approx \boxed{\$1,247}$$

principal loan

d.p.

Algebraically determine and state the down payment, rounded to the *nearest dollar* that Jim needs to make in order for his mortgage payment to be \$1100. $M = 1100$ d.p. $= 172,600 - P$

$$1100 = P \cdot \frac{(.00305(1.00305)^{180})}{(1.00305)^{180} - 1}$$

$$1100 = P \cdot (.0072276558\ldots)$$

$$P = \frac{1100}{.0072276558\ldots} = 152193.1906$$

$$\text{d.p.} = 172,600 - 152,193.1906$$

$$\text{d.p.} = 20,406.80935$$

$$\boxed{\text{d.p.} \approx \$20,407}$$