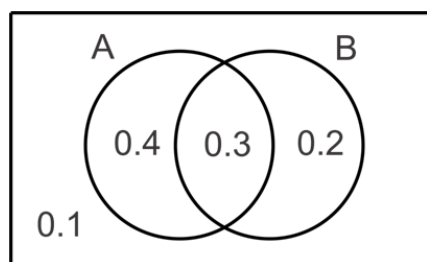


*For this whole unit,
read the next day's
notes to get familiar
with the context of
each problem.*

Unit 12 Probability



You will study:

- **Basic Probability (Day 1)**
- **Probability Using Two-Way Tables (Day 2)**
- **Calculate Probability Using Two-Way Tables (Day 3)**
- **Conditional Probability Using Two-Way Tables (Day 4)**
- **Conditional Probability & Independence (Day 5)**
- **More Conditional Probability, Independence & Venn Diagrams (Day 6)**
- **Events & Venn Diagrams (Day 7)**
- **More Venn Diagrams, Complement & Conditional Formula (Day 8)**
- **Multiplication Rule (Day 9)**
- **Probability Interpretation, Addition Rule & Disjoint Events (Day 10)**

Day 1 Basic Probability

Tree Diagrams

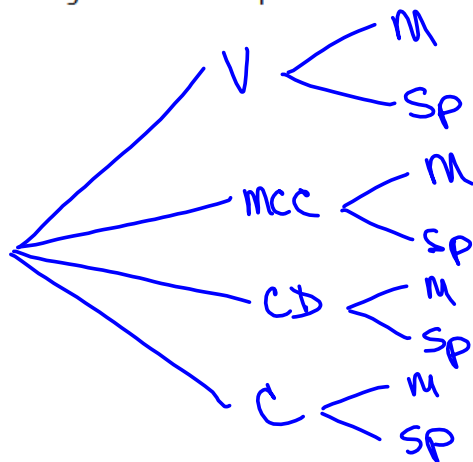
A **tree diagram** is a way to list all possible outcomes in a sample space for two or more activities. We can determine how many possible outcomes there are by counting.

1. At the Banana Boat ice-cream store, there are four possible flavors of ice-cream:

Vanilla, Mint Chocolate Chip, Cookie Dough, Chocolate

There are two possible toppings: Marshmallows or Sprinkles.

- a. Draw a tree diagram to show the possible sundaes that can be made.



- b. List the sample space for the possible sundaes.

$\{(v, m), (v, sp), (mcc, m), (mcc, sp),$
 $(cd, m), (cd, sp), (c, m), (c, sp)\}$

- c. How many sundaes were possible? *1 ice cream + 1 topping*

8

4×2

- d. Are the outcomes in the sample space equally likely? Explain your answer.

No. Flavors are chosen by people not at random.

In any population, flavor choices are not likely equal.

Counting Principle → If one choice can occur in any of m ways and a second choice can occur in any of n ways, then the total number of ways both can occur is $m \cdot n$.

and

2. Ellen has 4 pairs of pants, 3 shirts, and 3 scarves that all match. How many outfits that include pants, a shirt, and a scarf can she put together?

$$4 \text{ pants} \times 3 \text{ shirts} \times 3 \text{ scarves} = 36 \text{ outfits}$$

3. A delicatessen serves sandwiches on rye bread, white bread, or rolls. There are eight different sandwich fillings. Customers have a choice of lettuce or no lettuce on the sandwich. How many different sandwiches can be assembled?

$$3 \text{ breads} \times 8 \text{ fillings} \times 2 \text{ lettuce} = 48$$

4. There are 4 entrances to a building, 3 escalators up, and 2 escalators down. How many ways can a person enter from outside, go to the second floor, return to the first floor, and then leave the building?

$$4 \times 3 \times 2 \times 4 = 96$$

5. A six-sided die and a fair coin are tossed together. How many outcomes are in the sample space?

$$6 \times 2 = 12$$

6. How many 7-digit telephone numbers can be created if the first digit cannot be 0?

0 → 9 ~~10~~ ¹5

$$\frac{9}{1-9} \times \frac{10}{0-9} \times \frac{10}{10} \times \frac{10}{10} \times \frac{10}{10} \times \frac{10}{10} \times \frac{10}{10}$$
$$9,000,000$$

Probability

A. Basics of Probability

1. Sample Space \rightarrow the set of all possible ways in which a probability experiment can turn out.
 - a. Biased Sample \rightarrow one or more parts of the population are favored over others
 - b. Unbiased Sample \rightarrow every possible sample has an equal chance of being selected

2. Favorable Outcomes (successes) \rightarrow the number of outcomes that will make an event occur.

Notation: $P(E) = \frac{\text{number of favorable outcomes}}{\text{total possible outcomes}}$

$P(E) \rightarrow$ probability of an event

$$P(\text{Rain}) = \frac{\text{\# of rainy days}}{\text{total \# of days}}$$

Prob. of it raining

3. Some probability facts:

a. $P(E)$ ranges from 0 to 1b. $P(E) = 0$ if event E is impossiblec. $P(\text{not } E) = 1 - P(E)$ d. $P(E) = 1$ is event E is certain
100%

e. If events A and B have no successful outcomes in common, then

$$P(A \text{ or } B) = P(A) + P(B)$$

(called mutually exclusive or disjoint events)

$$P(\text{Red and club}) = 0$$

f. If events A and B have outcomes in common that are successes for both events, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(\text{Red or diamond}) = P(\text{Red}) + P(\text{diamond}) - P(\text{Red and diamond})$$

g. $P(A \text{ and } B) = P(A) * P(B)$

- B. Theoretical Probability → what is expected to occur in an experiment. What **should** happen.
- C. Experimental Probability → estimated from observed simulations or experiments. What **actually** happened.

Examples Using Theoretical Probability

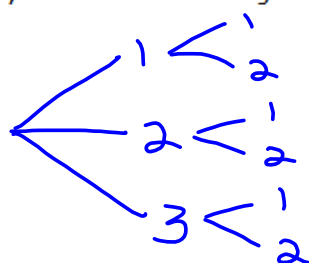
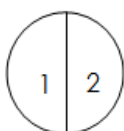
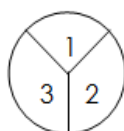
1. A jar contains 2 red marbles, 3 white marbles, and 4 black marbles. If one marble is chosen at random, find

a. $P(\text{red}) = \frac{2}{9}$ b. $P(\text{red or white}) = \frac{5}{9}$

c. $P(\text{green}) = 0$ d. $P(\text{not red}) = \frac{7}{9}$
 $1 - \frac{2}{9} = \frac{7}{9}$

- Many Stage Experiments:**
1. tossing 2 coins
 2. tossing 2 die
 3. tossing a coin and tossing a die

2. Consider spinning each spinner once. The sample space may be shown in a tree diagram:



There are a total of 6 outcomes. 3×2

Find the probability of

a. $P(3, 2) = \frac{1}{6}$

b. $P(\text{both odd}) = \frac{2}{6}$

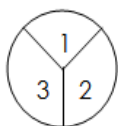
c. $P(\text{at least one even}) = \frac{4}{6}$

d. $P(\text{at most one even}) = \frac{5}{6}$

e. $P(\text{odd, even}) = \frac{2}{6}$

0, 1 E

3. Based on the chance experiment of spinning the spinner and selecting a card from the bag:



Blue A	Blue B	Blue C	Blue D	Red E	Red F
-----------	-----------	-----------	-----------	----------	----------

$$3 \times 6 = 18$$

Find the probability of

1. spinning an odd number.
2. spinning an odd number and selecting a red card from the bag.
3. spinning an odd number and selecting a blue card from the bag.
4. spinning an even number or selecting a red card from the bag.
5. not selecting a blue card from the bag.

	Outcomes	Probability
1.	1 A-F (4) 3 A-F (6)	$\frac{10}{18}$ or $\frac{5}{9}$
2.	1 E, F (2) 3 E, F (6)	$\frac{8}{18}$
3.	1 A-D (4) 3 A-D (6)	$\frac{10}{18}$
4.	2 A-F (4) 1 E, F (2) 3 E, F (6)	$\frac{12}{18}$
5.	1 E, F (2) 2 E, F (4) 3 E, F (6)	$\frac{10}{18}$

How do the ideas of probability help you make decisions?

It estimates the #times an outcome should occur and helps us to make better decisions.

