1-12 HW Answer Key

1 & 2. Look at next page

3ab. {10, -4}

4ab. $\left\{-7 \pm \sqrt{57}\right\}$

5. $\left\{-3 \pm \sqrt{29}\right\}$

6. $\left\{2 \pm \sqrt{10}\right\}$

7. {0, -1, 1/2}

8. (2x+1)(x+2)(x-2)

9. $\left\{\dfrac{7 \pm \sqrt{69}}{2}\right\}$

1. Describe the process of completing the square.
Get the equation in the form of $x^2+bx =$ constant.
Divide b by 2, square it and add to both sides.
Factor left side.
Take square root of both sides.
Solve for x.

2. When do we use this method?
We use this when we have a quadratic that does not factor as a perfect square.
Easier if 'b' term is even.
Want 'a' term to be equal to 1.

For the following: a) solve by completing the square and b) solve using the quadratic formula. by finding the discriminant first.

3. $x^2 - 6x = 40$

a) $x^2-6x+9 = 40+9$
$\sqrt{(x-3)^2} = \pm\sqrt{49}$
$(x-3) = \pm 7$
$x = 3 \pm 7$
$x = 3+7$, $x = 10$ | $x = 3-7$, $x = -4$
$\{10, -4\}$

b) $x^2-6x-40=0$ (a, b, c)
disc $= (-6)^2-4(1)(-40)$
$= 36+160 = 196$
$x = \dfrac{-(-6) \pm \sqrt{196}}{2(1)}$
$x = \dfrac{6 \pm 14}{2} = 3 \pm 7$
$\{10, -4\}$

$\left(\frac{b}{2}\right)^2$

4. $x^2 + 14x - 8 = 0$

a) $x^2+14\cancel{x}+49 = 8+49$
$\sqrt{(x+7)^2} = \pm\sqrt{57}$
$x+7 = \pm\sqrt{57}$
$x = \{-7 \pm \sqrt{57}\}$

b) $x^2+14x-8=0$ (a, b, c)
disc $= (14)^2-4(1)(-8)$
$= 196+32 = 228$
$x = \dfrac{-14 \pm \sqrt{228}}{2(1)} = \dfrac{-14 \pm \sqrt{4 \cdot 57}}{2}$
$x = \dfrac{-14 \pm 2\sqrt{57}}{2} = \{-7 \pm \sqrt{57}\}$

Solve by completing the square only.

5. $\frac{12x + 2x^2}{2} = \frac{40}{2}$

$x^2 + 6x = 20$

$x^2 + 6x + 9 = 20 + 9$

$\sqrt{(x+3)^2} = \pm\sqrt{29}$

$x + 3 = \pm\sqrt{29}$

$x = \{-3 \pm \sqrt{29}\}$

6. $\frac{5x^2 - 20x}{5} = \frac{30}{5}$

$x^2 - 4x + 4 = 6 + 4$

$\sqrt{(x-2)^2} = \pm\sqrt{10}$

$x - 2 = \pm\sqrt{10}$

$x = \{2 \pm \sqrt{10}\}$

7. Solve by factoring: GCF | Long P/S

$4x^3 + 2x^2 - 2x = 0$

P = -2
S = 1
-1, 2

$2x(2x^2 + x - 1) = 0$

$2x(2x^2 + 2x - x - 1) = 0$

$2x[2x(x+1) - 1(x+1)] = 0$

$2x(x+1)(2x-1) = 0$

$2x = 0$	$x + 1 = 0$	$2x - 1 = 0$
$x = 0$	$x = -1$	$x = 1/2$

$\{0, -1, 1/2\}$

8. Factor $2x^3 + x^2 - 8x - 4$

$= x^2(2x+1) - 4(2x+1)$

$= (2x+1)(x^2 - 4)$

$= (2x+1)(x+2)(x-2)$

$-\frac{7}{2} \cdot -\frac{7}{2} = \frac{49}{4} = P$ ✓ $-\frac{7}{2} + -\frac{7}{2} = \frac{-14}{2} = -7 = \text{sum}$ ✓

9. In the notes, we stated that completing the square is not "pretty" when the 'x' term is odd. Solve the following by completing the square, and then explain why it is not "pretty". $\left(\frac{b}{2}\right)^2 = \left(\frac{-7}{2}\right)^2 = \frac{49}{4}$

$x^2 - 5 = 7x$

$x^2 - 7x + \boxed{\frac{49}{4}} \cancel{+5} + \frac{49}{4} = \frac{20}{4} + \boxed{\frac{49}{4}}$

$\frac{5}{1} \cdot \frac{4}{4}$

$(x + \frac{b}{2})^2$

$\sqrt{(x - 7/2)^2} = \pm\sqrt{\frac{69}{4}}$

$x - 7/2 = \pm\frac{\sqrt{69}}{2}$

$x = \left\{\frac{7}{2} \pm \frac{\sqrt{69}}{2}\right\}$ or $\left\{\frac{7 \pm \sqrt{69}}{2}\right\}$

It's not "pretty" because we are adding a fraction.

Helpful chart in your packet with examples: review on your own.

Choosing which method to use to solve quadratics takes as much skill as being able to use these methods. This chart may help.

Summary of Solving Quadratic Equations

Method	When to Use	Examples
Factoring	$c = 0$ or the expression is easily factorable.	$x^2 + 4x + 3 = 0$ $(x + 3)(x + 1) = 0$ $x = -3$ or $x = -1$
Square roots $x^2 - 15 = 0$ $x^2 + 0x - 15 = 0$	The variable side of the equation is a perfect square.	$(x - 5)^2 = 24$ $\sqrt{(x-5)^2} = \pm\sqrt{24}$ $x - 5 = \pm 2\sqrt{6}$ $x = 5 \pm 2\sqrt{6}$
Completing the square	$a = 1$ and b is an even number.	$x^2 + 6x = 10$ $x^2 + 6x + \square = 10 + \square$ $x^2 + 6x + \left(\frac{6}{2}\right)^2 = 10 + \left(\frac{6}{2}\right)^2$ $(x + 3)^2 = 19$ $x = -3 \pm \sqrt{19}$
Quadratic Formula	Numbers are large or complicated, and the expression does not factor easily.	$5x^2 - 7x - 8 = 0$ $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(5)(-8)}}{2(5)}$ $x = \frac{7 \pm \sqrt{209}}{10}$

Today- you chose the method to solve 1-6 on notes 1-13.

Which method do you like the best? Why?

1) $\{-6, 8\}$

2) $\{0, -4, \pm 3\}$

3) $\frac{2 \pm \sqrt{7}}{3}$

4) $3 \pm \sqrt{47}$

5) $4 \pm \sqrt{29}$

6) $\{0, -6 \pm \sqrt{2}\}$

1-13: Solving all types of Quadratic Equations

Decide on the method (factoring, square roots, completing the square, or the quadratic formula) and then solve. Use each method at least once.

1. $x^2 - 2x - 48 = 0$

$P = -48, \quad S = -2 \qquad 6, -8$

$(x+6)(x-8) = 0$

$x = -6 \mid x = 8$

$\{-6, 8\}$

2. $3x^4 + 12x^3 - 27x^2 - 108x = 0$

$3x(x^3 + 4x^2 - 9x - 36) = 0$

$x^2(x+4) - 9(x+4) = 0$

$(x+4)(x^2 - 9) = 0$

$3x(x+4)(x+3)(x-3) = 0$

$3x = 0 \mid x+4 = 0 \mid x+3 = 0 \mid x-3 = 0$

$x = 0 \mid x = -4 \mid x = -3 \mid x = 3$

$\{0, -4, \pm 3\}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

3. $3x^2 - 4x = 1 \rightarrow \overset{a}{3x^2} \overset{b}{-4x} \overset{c}{-1} = 0$

$x = \frac{4 \pm \sqrt{16 - 4(3)(-1)}}{2(3)} = \frac{4 \pm \sqrt{16+12}}{6} = \frac{4 \pm \sqrt{28}}{6} = \frac{4 \pm \sqrt{4}\sqrt{7}}{6}$

$x = \frac{4 \pm 2\sqrt{7}}{6} \div 2 = \left\{ \frac{2 \pm \sqrt{7}}{3} \right\}$

4. $\sqrt{(x-3)^2} = \pm\sqrt{47}$

$(x-3) = \pm\sqrt{47}$

$x - 3 = \pm\sqrt{47}$

$+3 \quad +3$

$x = \{3 \pm \sqrt{47}\}$

$\sqrt{m^2} = \pm\sqrt{47}$

$m = \pm\sqrt{47}$

~~$x^2 - 8x - 13 = 0$~~ $\left(\frac{b}{2}\right)^2 = \left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

5. $x^2 - 8x = 13$

$x^2 - 8x + \boxed{16} = 13 + \boxed{16}$

$\sqrt{(x-4)^2} \quad \pm\sqrt{29}$

$x - 4 = \pm\sqrt{29}$

$x = \{4 \pm \sqrt{29}\}$

$P = 34,\ S = 12$

1, 34 can't
2, 17 factor

6. $2x^4 + 24x^3 + 68x^2 = 0$

$2x^2(x^2 + 12x + 34) = 0$

$\left(\frac{12}{2}\right)^2 = 6^2 = 36$

$\frac{2x^2}{2} = \frac{0}{2}$

$\sqrt{x^2} = \sqrt{0}$

$x = 0$

Complete the square

$x^2 + 12x + \boxed{36} = -34 + \boxed{36}$

$\sqrt{(x+6)^2} = \pm\sqrt{2}$

$x + 6 = \pm\sqrt{2}$

$x = -6 \pm \sqrt{2}$

$\{0, -6 \pm \sqrt{2}\}$

What is the difference between factoring and solving for x?

Factoring is breaking down an expression into factors.
Solve for x is getting a value for x.
Factoring sometimes helps us to solve for x.

Challenge question.
A quadratic in standard form is $ax^2 + bx + c = 0$. We can use completing the square in order to derive the quadratic formula. Start by dividing each term by a.

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x + \boxed{} = -\frac{c}{a} + \boxed{}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{-4ac + b^2}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$