

Unit 2

Rational Expressions & Equations

You will learn to:

- Simplify rational expressions
- Add and subtract rational expressions
- Solve rational equations
- Multiply and divide rational expressions
- Simplify complex fractions

Rational Expressions

A rational expression is an expression that can be written as $\frac{P}{Q}$

where P and Q are polynomials and Q is not equal to zero.

anything divided by zero = undefined

Example: $\frac{x^2 - 4x}{x + 1}$

When identifying values for which a rational expression is undefined, identify the values of the variable that make the original denominator = 0.

$$x + 1 = 0$$

$$x = -1$$

makes our
fraction undefined

$$x + 1 \neq 0$$

$\{x \mid x \neq -1\} \leftarrow$ domain restriction

$$D=0$$

Identify any x-values for which the expression is undefined:

1. $\frac{4}{x-3}$ $x-3=0$
 $x=3$
DR: $\{x | x \neq 3\}$

2. $\frac{5}{x+4}$ $x+4=0$
 $x=-4$
DR: $\{x | x \neq -4\}$

3. $\frac{6x}{3x-1}$ $3x-1=0$
 $3x=1$
 $x=\frac{1}{3}$
DR: $\{x | x \neq \frac{1}{3}\}$

4. $\frac{2}{x^2-9}$ $x^2-9=0$
 $(x-3)(x+3)=0$
 $x=3 | x=-3$
DR: $\{x | x \neq \pm 3\}$

Simplifying Rational Expressions

- A. Completely **factor** numerator and denominator
- B. Divide common ~~terms~~ **factors**
- C. Look for a factor of -1 (ie: $\frac{5-3}{3-5} = -1$ so.... $\frac{a-b}{b-a} = -1$)

Simplify. Identify any values for which the expression is undefined. (Write as domain restrictions)

5. $\frac{10x^4}{20x^2} = \frac{x^2}{2}$
 No + or - signs, so no factoring

$\frac{20x^2}{20} \neq 0$
 $\sqrt{x^2} \neq 0$
 $x \neq 0$

7. $\frac{x^2 - 4x}{x^2 - 2x - 8} = \frac{x(x-4)}{(x-4)(x+2)} = \frac{x}{x+2}$
 $\{x | x \neq 0\}$

$\frac{x}{x+2}$
 $\{x | x \neq -2\}$

$x+2$
 "glued" together

6. $\frac{x+3}{x^2 - 2x - 15} = \frac{x+3}{(x-5)(x+3)} = \frac{1}{x-5}$
 $x \neq 5$ $x \neq -3$

8. $\frac{5x^2 - 15x}{27x - 3x^3} = \frac{5x(x-3)}{3x(9-x^2)} = \frac{5x(x-3)}{3x(3-x)(3+x)} = \frac{-5}{3(3+x)}$
 $\{x | x \neq 0, \pm 3\}$

$$9. \frac{\frac{2(x^2-25)}{2x^2-50}}{x^2+8x+15} = \frac{2(x-5)(x+5)}{(x+3)\cancel{(x+5)}} \\ \{x | x \neq -3, -5\} \\ = \boxed{\frac{2(x-5)}{x+3}}$$

$$10. \frac{10-5x}{15x-30} = \frac{\cancel{5}(2-x)}{3\cancel{5}(x-2)} = \frac{-1}{3} \\ \{x | x \neq 2\}$$

Determine whether or not the rational expressions $\frac{x+4}{(x+2)(x-3)}$ and $\frac{(x+4)\cancel{(x+1)}}{(x+1)(x+2)(x-3)}$ are equivalent for $x=-1$, $x=-2$ and $x=3$. Explain your results.

↑ both undefined
 → The first expression is defined, but the second is not.

Adding or Subtracting With Like Denominators:

- Add the numerators - Keep the denominators
- Factor if possible
- Simplify if possible

Examples: $\frac{3}{10} + \frac{6}{10} = \frac{9}{10}$ so... $\frac{3x}{10} + \frac{6x}{10} = \frac{9x}{10}$

Add or Subtract:

1. $\frac{9y}{2x} - \frac{5y}{2x} = \frac{2y}{2x} = \boxed{\frac{y}{x}}$
 $x \neq 0$

2. $\frac{3x-4}{x+3} + \frac{2x+5}{x+3} = \boxed{\frac{5x+1}{x+3}}$
 $x \neq -3$

3. $\frac{x^2+2x}{x+1} + \frac{1}{x+1} = \frac{x^2+2x+1}{x+1}$
 $x \neq -1$
 $= \frac{(x+1)(x+1)}{(x+1)}$
 $= \boxed{x+1}$

4. $\frac{9x-2}{3x-5} - \frac{3x+8}{3x-5} = \frac{6x-10}{3x-5} = \frac{2(3x-5)}{3x-5} = \boxed{2}$
 $3x-5 \neq 0$
 $3x \neq 5$
 $x \neq \frac{5}{3}$

or $\frac{(9x-2)-(3x+8)}{3x-5} = \frac{9x-2-3x-8}{3x-5}$

Discuss with your partner:

- a) How do you reduce a rational expression to lowest terms?
- b) How do you know which values of the variable(s) to exclude for a rational expression?