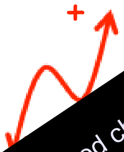
HW 5-4 Answers

1. They are imaginary (covered in unit 4)
2. There are 4 real zeros
3. The power tells you the number of zeros, so there are “m”
4. $(x + 5)(x^2 - 5)$
5. $(1 + 3x)(1 - 3x + 9x^2)$

6. a. 4
   b. -
   c. 

7. a. 7
   b. +
   c. 

8 - 11 See next page

12. $P(x) = x^2(x + 5)(x - 3)$
13. $P(x) = x(x + 2)(x - 2)$

When you have finished checking your homework answers please do the warm-up at the top of today's notes.

1. The graph of a polynomial function never passes through the x-axis but passes through the y-axis once. What does that tell you about the zeros of the graph?

they are imaginary (covered in unit 4)

2. The graph of a polynomial function passes through the x-axis four times and the y-axis once. What does this tell you about the zeros of the graph?

there are 4 real zeros

3. Consider the polynomial $P(x) = Ax^m + Bx$. What can you determine about the number of zeros from the equation?

the power tells you the number of zeros, so there are "m"

In 4 & 5, factor:

4. $x^3 + 5x^2 - 5x - 25$

$x^2(x+5) - 5(x+5)$

$= (x+5)(x^2-5)$

5. $1 + 27x^3$

$a = 1 \quad b = 3x$

$(1+3x)(1-3x+9x^2)$

Without your calculator:

a. state degree
b. state the sign of the leading coefficient
c. sketch (no graph paper) the end behavior

6. $P(x) = -2x^4 + 4x^3 - 2x + 7$

a. 4
b. −

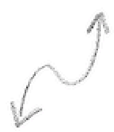
$\}$ − even

c. 

7. $P(x) = 4x^7 + 2x^3 - 5x$

a. 7
b. +

$\}$ + odd

c. 

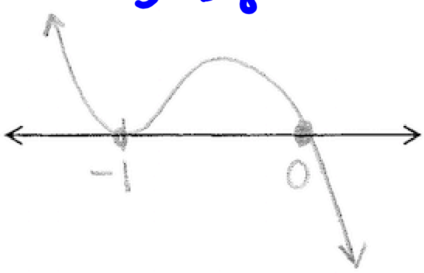
Find the zeros of each polynomial, state the multiplicity of each. Sketch (including the end behavior) - no calculators!

8. $P(x) = -x(x + 1)^2$

Z	M	T/C
−1	2	T
0	1	C

− odd

3 = Degree

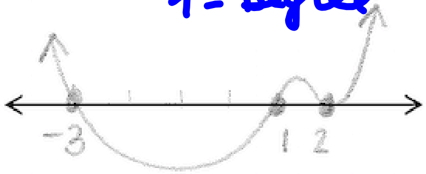


9. $Q(x) = (x + 3)(x - 1)(x - 2)^2$

Z	M	T/C
−3	1	C
1	1	C
2	2	T

+ even

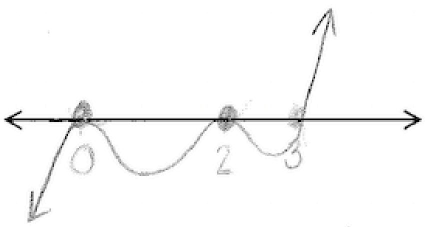
4 = Degree



10. $R(x) = +x^2(x - 2)^2(x - 3)$

Z	M	T/C
0	2	T
2	2	T
3	1	C

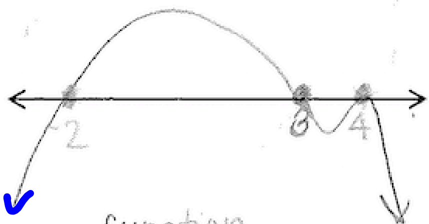
+ odd



11. $M(x) = -(x + 2)^3(x - 3)(x - 4)^2$

Z	M	T/C
−2	3	C
3	1	C
4	2	T

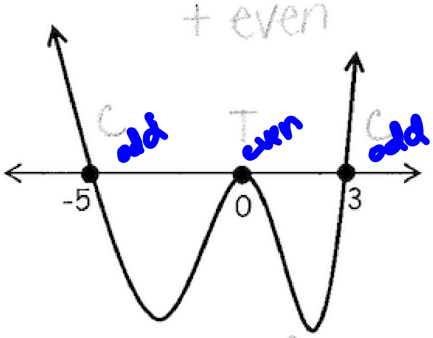
− even



function

Given the following graphs, write a possible polynomial equation for the graph.

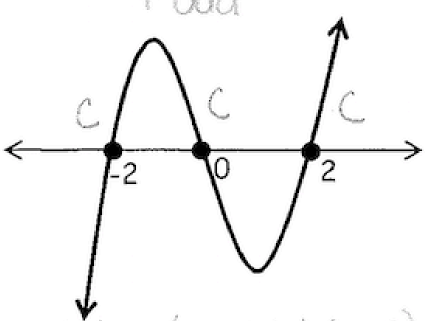
12. + even



$P(x) = (x+5)(x)^2(x-3)$

$P(x) = x^2(x+5)(x-3)$

13. + odd



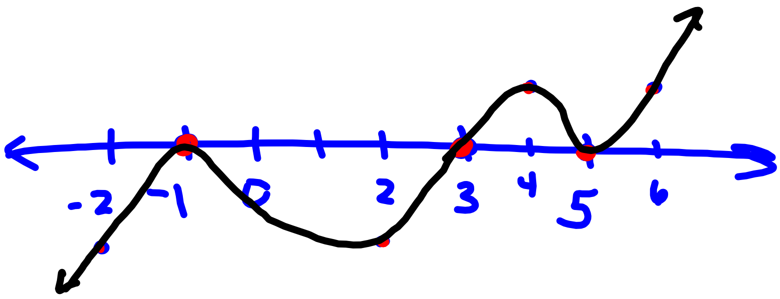
$P(x) = (x+2)(x)(x-2)$

$P(x) = x(x+2)(x-2)$

Day 5

Factoring & Solving

Warm-Up:
A function f has zeros at -1, 3, and 5. We know that f(-2) and f(2) are negative, while f(4) and f(6) are positive. Sketch a graph of f.



$$f(x) = k(x+1)^2 (x-3)^1 (x-5)^2$$

sometimes $K>0$

You have been given a set of problems.
The directions for some say, "factor" whereas others say, "solve".
What's the difference between the two? How would you expect your answers to look?

1. factor expressions (Not solve) ans: ( )( )
2. solve equations (sometimes by factoring) { , }

Factor completely each of the following:

1. $x^8 - 1$ DOTS

$(x^4+1)(x^4-1)$

$(x^4+1)(x^2+1)(x^2-1)$

$\boxed{(x^4+1)(x^2+1)(x+1)(x-1)}$

2. $x^4 - 2x^2 + 1$

$(x^2-1)(x^2-1)$

$(x+1)(x-1)(x+1)(x-1)$

$\boxed{(x+1)^2(x-1)^2}$

(hint: be careful with this one!)

3. $64x^6 - 1$ Both! Do DOTS 1st!

$(8x^3+1)(8x^3-1)$ $a = \sqrt[3]{8x^3} = 2x$

$b = \sqrt[3]{1} = 1$

$\boxed{(2x+1)(4x^2-2x+1)(2x-1)(4x^2+2x+1)}$

4. $2x^5 + x^4 + 2x^3 + x^2$

$x^2(2x^3+x^2+2x+1)$

$x^2[x^2(2x+1)+1(2x+1)]$

$\boxed{x^2(2x+1)(x^2+1)}$

5. $\dfrac{x^{5n}}{x^{2n}} + \dfrac{x^{2n}}{x^{2n}}$

$a^2 = (x^n)^2 = x^{2n}$

$x^{2n}(x^{3n}+1)$

$a = \sqrt[3]{x^{3n}} = x^n$

$b = \sqrt[3]{1} = 1$

$(a+b)(a^2 - ab + b^2)$

$\boxed{x^{2n}(x^n+1)(x^{2n} - x^n + 1)}$

6. $2(x + 2)^2 + (x + 2) - 3$

Let $u = x+2$

$2u^2 + u - 3$

$(2u+3)(u-1)$

$(2(x+2)+3)(x+2-1)$

$(2x+4+3)(x+1)$

$\boxed{(2x+7)(x+1)}$

7. $25x^{2n} - 625$

$25(x^{2n} - 25)$

$25(x^n - 5)(x^n + 5)$

All of the previous problems were factorable. If we set each of them equal to 0, only some are solvable. Why?

#5 and #7 were Not solvable because they have 2 variables (also one variable in exponent)

Solve each of the following (factor completely first):

1. $4x^5 - 8x^3 + 4x = 0$

$4x(x+1)^2(x-1)^2 = 0$

$4x=0$	$x+1=0$	$x-1=0$
$x=0$	$x=-1$	$x=1$

$\{0, \pm 1\}$

2. $x^6 - 16x^2 = 0$

$x^2(x^2+4)(x+2)(x-2)=0$

$x^2=0$	$x^2+4=0$	$x+2=0$	$x-2=0$
$x=0$	$\sqrt{x^2}=\sqrt{-4}$	$x=-2$	$x=2$
	$x=\pm 2i$		

$\{0, \pm 2i, \pm 2\}$

3. $x^4 - 13x^2 + 36 = 0$

$(x+2)(x-2)(x+3)(x-3)=0$

$x+2=0$ $x=-2$	$x=2$	$x=-3$	$x=3$

$\{\pm 2, \pm 3\}$

4. $3x^4 - 24x = 0$

$3x(x-2)(x^2+2x+4)=0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$3x=0$ $x=0$	$x-2=0$ $x=2$	$x^2+2x+4=0$ $x = \frac{-2 \pm \sqrt{-12}}{2(1)} = \frac{-2 \pm i\overset{2}{\cancel{\sqrt{4}}}\sqrt{3}}{2}$ $x = \frac{-2 \pm 2i\sqrt{3}}{2} = -1 \pm i\sqrt{3}$

b^2-4ac

$4-4(1)(4)=-12$

$\{0, 2, -1 \pm i\sqrt{3}\}$