Sketch both #29 & 30 on tonight's homework.

Polynomial Functions

Put away your

calculators NOW

Oct 19-7:25 PM

Polynomial Functions

Polynomial in 1 variable:

$$a_o x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots a_{n-1}x + a_n$$

where coefficients represent complex numbers and $a_o \neq 0$, $n \in N$ (Natural #s) 1, 2, 3, ...

The degree of the polynomial is the greatest exponent of the variable.

The leading coefficient is the one with the degree variable.

Oct 18 - 7:31 AM

$3x^3 + 5x^2 + 7$ degree = 3

leading coefficient = 3

$2w^2 + 7w - 4w^5 + w^4$ degree = 5

leading coefficient = -4

Oct 18 - 8:49 AM

Fundamental Theorem of Algebra

Every polynomial equation with degree > 0 has at least 1 root in the complex numbers.

real #

imag #

Every polynomial P(x) of degree n (n > 0) can be written as the product of a constant k ($k \neq 0$) and **n** linear factors.

Oct 18 - 7:31 AM

$$P(x) = k(x - r_1)(x - r_2)\ldots(x - r_n)$$

$\therefore$ A polynomial equation of degree n has exactly n complex roots.

K usually = 1 $r_1, r_2, r_3\ldots$ are the roots

1. $P(x) = x^2 - x - 6$ → 2 factors 2 roots

$P(x) = (x-3)(x+2)$

2. $P(x) = x^4 - 81$ → 4 factors 4 roots

$P(x) = (x^2+9)(\underline{x^2-9})$

$P(x) = (x+3)(x-3)(x+3i)(x-3i)$

$-9i^2 = +9$

Oct 18 - 7:33 AM

Determine whether each polynomial is a polynomial in 1 variable. If yes, state the degree. If no, tell why not.

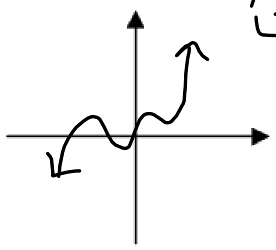
1. $x^2 + 3xy - 5y^3$ → no, it has 2 variables
2. $x^2 - x^3 - x + 3x^4 - 1$ → yes, degree = 4
3. $8 - x - 4x^2 + \frac{7}{x}$ → no, x in denom, $7x^{-1}$ → expo is not a natural #
4. $a^3 + 2a + \sqrt{3}$ → yes, degree = 3
5. $\frac{1}{x} = \frac{1}{2x}$ → No, x's are in denom. Negative exponents
6. $2i\sqrt{x} + ix^8 + 5x^4 + 9$ → no, x under radical = $x^{1/2}$, $1/2$ is not a natural #

Oct 12-8:05 PM

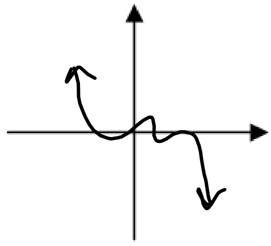
General Shapes of Polynomial Functions of Higher Degree have these same basic characteristics:

4 GENERAL SHAPES: a_0 = lead coeffic, n = degree

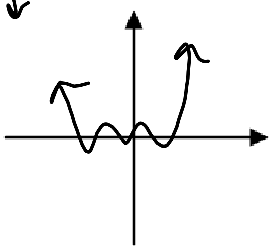
① + a_0 is > 0 and n is odd $+x^{odd}$ → right end ↑   odd → left end is opposite



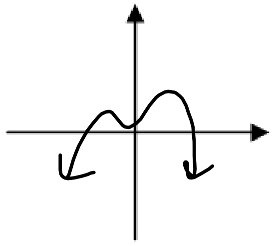
② − a_0 is < 0 and n is odd $-x^{odd}$



③ + a_0 is > 0 and n is even $+x^{even}$



④ − a_0 is < 0 and n is even $-x^{even}$



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Determine whether each number is a root of P(x) and describe the end behavior of the graph:

1. $P(x) = x^3 + 3x^2 - 3x + 6$ -3 → (-3,0)?

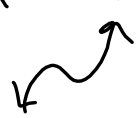
$P(-3) = (-3)^3 + 3(-3)^2 - 3(-3) + 6$

$= \cancel{-27} + \cancel{27} + 9 + 6$

$= 15 \neq 0$

No, not a root

$x^3 \rightarrow +x^{odd}$



2. $P(x) = x^4 - 4x^3 - x^2 + 4x$ 4

$P(4) = \cancel{4^4} - \cancel{4(4)^3} - 4^2 + 4(4)$

$= -16 + 16 = 0$

yes → root

$x^4 \rightarrow +x^{even}$

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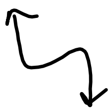
Determine whether each number is a root of P(x) and describe the end behavior of the graph:

3. $P(x) = -x^5 + 2x^2 - 3$ $\underline{-1}$

$P(-1) = -(-1)^5 + 2(-1)^2 - 3$
$= -(-1) + 2 - 3$
$= 1 + 2 - 3$
$= 0$
yes, root
$-x^5 \rightarrow -x^{odd}$



4. $P(x) = -x^6 + x^5 + 2x^3 - x + 5$ $\underline{2}$

$P(2) = -(2)^6 + 2^5 + 2(2)^3 - 2 + 5$
$= \underline{-64 + 32} + 16 + 3$
$= -32 + 19 = -13$
$\therefore$ NO, not a root
$-x^6 \rightarrow -x^{even}$

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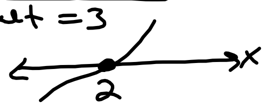
Multiplicity of Roots:

A factor of $\underline{(x-a)^k}$ yields a repeated zero $x = a$ of **multiplicity** k.

$(x-2)^k$ $(x-2)^3$ $x=2$, mult $=3$

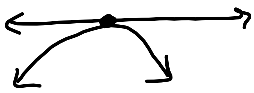
If k is **odd**, the graph **crosses** the x-axis at $x = a$.



$(x-2)^3$

If k is **even**, the graph **touches** (but does not cross) the x-axis at $x = a$.
(ie graph is tangent to the x-axis at a)

$(x-2)^4$ $x=2$, mult $=4$ even



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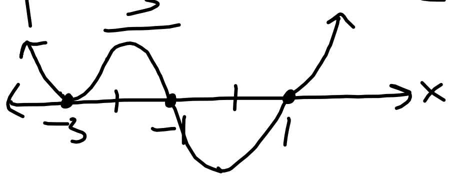
Find the zeros of each polynomials and state the multiplicity of each. Sketch.

1. $y = (x+3)^2(x+1)(x-1)^3$

even odd

Roots	m/k	$+x^6$	Touch/Cross
−3	2		T
−1	1		C
1	3		C

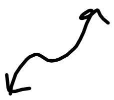


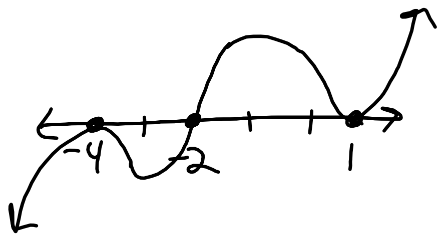
Find the zeros of each polynomials and state the multiplicity of each. Sketch.

$x^4 \cdot x \cdot x^2$

2. $y = (x+4)^4(x+2)(x-1)^2$

even odd

R	m	$+x^7$	T/C
−4	4		T
−2	1		C
1	2		T



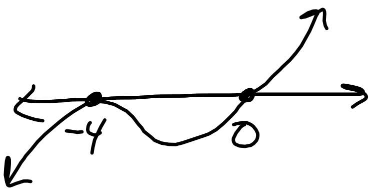
Find the zeros of each polynomials and state the multiplicity of each. Sketch.

3. $y = x^3 + 8x^2 + 16x$

$y = x(x^2 + 8x + 16)$

$y = x(x+4)(x+4)$

$y = x(x+4)^2$



R	m	$+x^3$	T/C
0	1		C
-4	2		T

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Homework:

pg 269 - 270:

12 - 18 even, 22

29, 30 sketch both

Graded Due Monday!

Oct 19-7:39 PM

Oct 23-4:48 PM