TIP on HW tonight: #2) variables t & n are in days, not years.
#3) Add 'annual' to question at end: What 'annual' interest rate did he charge you?

Applications of Logs

Nov 10-11:11 AM

Compounded Interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

Compounded Continuously: $A = Pe^{rt}$

P = principal (amount invested)
A = Amount after t years
e = Euler's number
r = rate as a decimal
t = number of years
n = number of compounds per year

Nov 10-9:00 AM

$=P$ $A = P(1+\frac{r}{n})^{nt}$ $r = .065$ $t = 3.5$

1. If \$5000 is invested at 6.5% annual interest rate, how much will you have after $3\frac{1}{2}$ years if the money is compounded:

a. Annually $n=1$

$A = 5000(1 + \frac{.065}{1})^{1(3.5)}$

$A = \$6232.95$

b. Quarterly $n=4$

$A = 5000(1 + \frac{.065}{4})^{4(3.5)}$

$A = \$6265.82$

c. Monthly

$n = 12$

$A = 5000(1 + \frac{.065}{12})^{12(3.5)}$

$A = \$6273.43$

d. Continuously

$A = Pe^{rt}$

$A = 5000e^{.065(3.5)}$

$A = \$6277.29$

Nov 10-9:02 AM

2. How long will it take your \$5000 invested at 6.5% to double compounded:

$r = .065$ $\quad \ln e = \log_e e = 1$

ISOLATE

a. Monthly $n = 12$

$A = P(1 + \frac{r}{n})^{nt}$

$\frac{2(5000)}{5000} = \frac{5000(1 + \frac{.065}{12})^{12t}}{5000}$

$2 = (1 + \frac{.065}{12})^{12t}$

$\log 2 = \log(1 + \frac{.065}{12})^{12t}$

$\log 2 = 12t \log(1 + \frac{.065}{12})$

$t = \frac{\log 2}{(12 \log(1 + \frac{.065}{12}))}$

$t = 10.69$ years

alternative

$\frac{\log_{(1 + \frac{.065}{12})}(2)}{12} = \frac{12t}{12}$

b. Continuously

$A = Pe^{rt}$

$2(5000) = 5000e^{.065t}$

$2 = 1e^{.065t}$

$\ln 2 = .065t \ln e$

$t = \frac{\ln 2}{.065}$

$t = 10.66$ years

Nov 27-2:49 PM

3. If you put \$5000 in an account that pays interest quarterly what interest rate must you receive in order to have \$7500 after 5 years?

$n = 4$ $\quad r$

$A = P(1 + \frac{r}{n})^{nt}$

$\frac{7500}{5000} = \frac{5000(1 + \frac{r}{4})^{4(5)}}{5000}$

$\sqrt[20]{1.5} = \sqrt[20]{(1 + \frac{r}{4})^{20}}$

$\sqrt[20]{1.5} - 1 = 1 + \frac{r}{4} - 1$

$4(\sqrt[20]{1.5} - 1) = (\frac{r}{4})4$

$r = 4(\sqrt[20]{1.5} - 1) = .08192 \approx 8.2\%$ tenth

Nov 10-9:03 AM

4. Mike invests $6000 in a retirement account with a fixed annual interest rate compounded continuously. After 15 years, his balance is $8099.15. What is the interest rate on the account?

$A=Pe^{rt}$

$\frac{8099.15}{6000}=\frac{6000e^{r(15)}}{6000}$

$\frac{8099.15}{6000}=e^{15r}$

$\ln\left(\frac{8099.15}{6000}\right)=15r\ln e$

$r=\frac{\ln\left(\frac{8099.15}{6000}\right)}{15}=.01999 \approx 2.0\%$

tenth & %

Nov 10-9:03 AM

The mass of a radioactive element at time t is given by

$$\boxed{A=A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}}$$

Where A_0 is the initial mass and h is the half-life of the element.

5. After 43 years, a 20-milligram sample of strontium-90 $\left({}^{90}Sr\right)$ decays to 6.071 mg. What is the half-life of strontium-90?

t, A_0, A, h?

$\frac{6.071}{20}=\frac{20(.5)^{43/h}}{20}$

$\frac{6.071}{20}=(.5)^{43/h}$

$h\log\left(\frac{6.071}{20}\right)=\left(\frac{43}{h}\log(.5)\right)h$

$h\log\left(\frac{6.071}{20}\right)=43\log(.5)$

$h=\frac{43\log(.5)}{\log\left(\frac{6.071}{20}\right)}=25.000$

25 years

Nov 10-9:03 AM

6. When a living organism dies, its carbon-14 decays. The half-life of carbon-14 is 5730 years. If the skeleton of a mastodon has lost 58% of its original carbon-14, when did the mastodon die? (to the nearest hundred years)

h

$A = A_0\left(\frac{1}{2}\right)^{t/h}$

$.42 = 1(.5)^{t/5730}$

$\log .42 = \frac{t}{5730} \log .5$

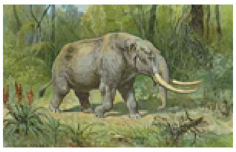
$5730 \log .42 = t \log .5$

$t = \frac{5730 \log .42}{\log .5} = 7171.3 \approx 7200$ years.

100 − 58

1 − .58

42% is remaining



Nov 10-9:03 AM

Nov 27-2:48 PM