

May 2016

Mathematics

Grade 6



Ratios and Proportional Relationships

Standard 1: Understand ratio concepts and use ratio reasoning to solve problems.

RP6.1.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

- reason how two quantities relate to each other
 - multiply, divide, justify and connect ratios and fractions to real world situations
- ratios are expressed as part-to-part or part-to-whole
- all fractions are only part-to-whole

RP6.1.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

- a/b division with decimals
- difference between a rate and a unit rate; unit rates are limited to non-complex fractions
 - *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar."*

RP6.1.3 Use ratio and rate reasoning to solve real-world and mathematical problems. (e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams or equations).

- make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables and plot the pairs of values on the coordinate plane. Use tables to compare ratios
- solve unit rate problems including those involving unit pricing and constant speed
 - *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
- find a percent of a quantity as a rate per 100; solve problems **involving** finding the whole, given a part and the percent
- use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities

RP6.1.4 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example: (BOCES Ex.)

- divide fractions
- simplify fractions
- understand rates and unit rates
- convert between lengths within metric and customary

The Number System

Standard 2: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

NS6.2.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

- know and create different physical and concrete models of fractions (e.g. models using recipes, fluids, numbers of people, sharing objects, cutting boards/food.)
- need to be able to interpret multi-step operations

Standard 3: Compute fluently with multi-digit numbers and find common factors and multiples.

NS6.3.1 Fluently divide multi-digit numbers using the standard algorithm

NS6.3.2 Fluently add, subtract, multiply and divide multi-digit decimals using the standard algorithm for each operation.

- fluent mastery of facts in all operations including with decimals

NS6.3.3 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

- *For example, express $36 + 8$ as $4(9 + 2)$*

Standard 4: Apply and extend previous understandings of numbers to the system of rational numbers.

NS6.4.1 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

NS6.4.2 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

- recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.
- understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

NS6.4.3 Understand ordering and absolute value of rational numbers.

- interpret statements of inequality as statements about the relative position of two numbers on a number line diagram
- write, interpret, and explain statements of order for rational numbers in real-world contexts.
- understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
 - Define absolute value and recognize the notation that is used for absolute value.
 - Compare absolute value of numbers foundation real world context relative to magnitude or size.
- distinguish comparisons of absolute value from statements about order.

NS6.4.4 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations

Standard 5: Apply and extend previous understandings of arithmetic to algebraic expressions.

EE6.5.1 Write and evaluate numerical expressions involving whole-number exponents.

- understand vocabulary: base, power, exponent, cubed, squared, standard notation, exponential notation, factor, etc.

EE6.5.2 Write, read and evaluate expressions in which letters stand for numbers.

- write expressions that record operations with numbers and with letters standing for numbers
 - know and understand vocabulary such as names for operations (sum, product, quotient, increased by, decreased by , less than, per, of)
- identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
- evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
 - substitute numbers for variables in basic expressions and formulas including fractions and decimals
 - know order of operations

EE6.5.3 Apply the properties of operations to generate equivalent expressions.

- know the number properties: commutative, associative, distributive, identify, inverse and zero.

EE6.5.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

EE6.5.5 Apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients.

- identify rational coefficients and like terms
- combine like terms, apply distributive property and its inverse (factoring)
- understand equivalent expressions through simplification

Standard 6: Reason about and solve one-variable equations and inequalities.

EE6.6.1 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

- recognize the following symbols: \leq \geq \neq $=$ $<$ $>$
- vocabulary: inequality, equation, variable, set, substitute, coefficient, constant, and operations

EE6.6.2 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

- understand that variables represent number values. For inequalities, variables can express more than one value.
- expressions do not have an equal sign

EE6.6.3 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.

- write equations to represent real life and mathematical problems
- use inverse operations to solve equations
- use substitutes to prove or check solutions

Standard 7: Use properties of operations to generate equivalent expressions.

EE6.7.1 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

- know the difference between an open and closed circle on a number line
- know that a closed circle is inclusive of a number and an open circle is exclusive of a number on a number line
- express infinite answers with the use of an arrow on a number line

Standard 8: Represent and analyze quantitative relationships between dependent and independent variables.

EE6.8.1 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

- know variables
- differentiate between independent and dependent variables
- know a T table

- identify parts of coordinate plane, such as an x-axis, y-axis, origin and all four quadrants

Geometry

Standard 9: Solve real-world and mathematical problems involving angle measure, area, surface area and volume.

G6.9.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

- when dividing into triangles, you start by composing triangles into rectangles to formulate the area of a triangle. Then using that knowledge, use the formula of area of a triangle to find the formula of any quadrilateral by dividing the quadrilateral into triangles and rectangles.
- understand the concept of area of a rectangle.
- identify and classify polygons (types of triangles, types of quadrilaterals)
- develop and apply the formulas for all triangles and quadrilaterals (square, rectangle, parallelograms, trapezoids)

G6.9.2 Find the volume of a right rectangular prism with **fractional or decimal** edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as it would be found in multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional or decimal edge lengths in the context of solving real-world and mathematical problems.

G6.9.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

G6.9.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

- understand the concept of area of a triangle and area of a rectangle.
 - naming 3-dimensional objects
- understand the vocabulary of three dimensional figures
 - faces, vertices, edges, net

Statistics and Probability

Standard 10: Develop understanding of statistical variability.

SP6.10.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.*

- understand the process of data collection, tally charts, and frequency tables

SP6.10.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape

- students need to know what a line plot is

SP6.10.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

- data collected can be described by the measures of central tendency (mean, median, mode, and range)

SP6.10.4 Display numerical data in plots on a number line, including dot plots, histograms and box plots.

- collect and organize data on a dot plot (line plot), histogram, and box plot

SP6.10.5 Summarize numerical data sets in relation to their context, such as by:

- report the number of observations
- describe the nature of the attribute under investigation, including how it was measured and its units of measurement
- give quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern (outliers) with reference to the context in which the data were gathered.
- relate the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered

Standard 11: Investigate chance processes and develop, use and evaluate probability models.

SP7.11.1 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

- understand the relationship between never, sometimes and always (0-) and the in-between (almost never, etc.)
- understanding of simple probability

SP7.11.2 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

○ *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times*

- predict and estimate outcomes based on Theoretical probabilities
- conduct experiments and collect data about the results

SP7.11.3 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

- develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events

- *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected*
- simple probability
- theoretical probability
- develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process
 - *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*
 - experimental probability
 - compare the experimental probability to the theoretical probability
 - use fractions, decimals and percentages to compare probability
 - law of large numbers

SP7.11.4 Find probabilities of compound events using organized lists, tables, tree diagrams and simulation.

- understand that, just as with simple events, the probability of a compound event is the fractions of outcomes in the sample space for which the compound event occurs
 - represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event, describe in everyday language (rolling double sixes), identify the outcomes in the sample space which compose the event
 - design and use a simulation to generate frequencies for compound events
 - *For example, use random digits as a simulation tool to approximate the answer to the question. If 40 percent of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*
 - create sample spaces-lists, tables, tree diagrams and simulations
 - given a sample space, identify the favorable outcomes of the event
 - given a sample space, determine the probability of a compound event
 - apply the Fundamental Counting Principle
 - distinguish between independent and dependent probability
 - make predictions based on probability experiments
- design and interpret a simulation

Standard 12: Use random sampling to draw inferences about a population.

SP7.12.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

- use sample data to gain information about a population
- make generalizations about a population
- determine the validity of a sample
- determine if random sampling does produce representative sample and support valid inferences

SP7.12.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

- describe certain characteristics of a population
- generate multiple samples using a given data set
- use the findings to compare estimate vs. actual and explain results

Standard 13: Draw informal comparative inferences about two populations.

SP7.13.1 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

- *for example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable*
- comparing two unequal distributions of data using a number of data points, measures of central tendency, shape, and variability (numerical data), and two-way tables (categorical variables). Using box plots, interquartile range, mean, and absolute deviation, range, and the concept of outliers to characterize the distribution (variability) of univariate data. Supporting claims about the results of investigations

SP7.13.2 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

- *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book*
- students will be able to calculate mean, median, range and deviation from data