Homework 10-8: Dilations of Lines

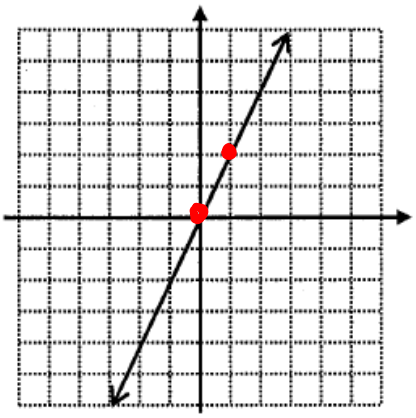
1. A dilation leaves a line passing through the center of dilation unchanged
2. A dilation takes a line NOT passing through the center of dilation to a parallel line.

Dilate the following lines for the given center and scale factor. Then, write the equation of the original AND the new line.

3. $D_{O,3}$

original:
$m = 2$
$b = 0$

new:
$m = 2$
$b = 3(0) = 0$



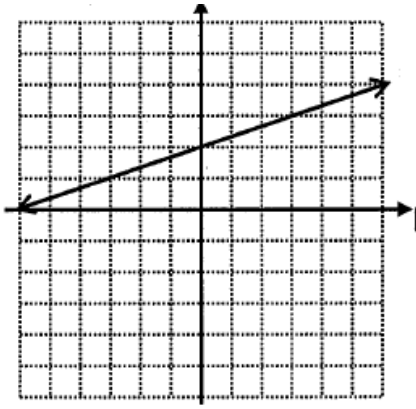
Equation: $y = 2x$

New Equation: $y = 2x$

4. D0, 2

original:
$m = \frac{1}{3}$
$b = 2$

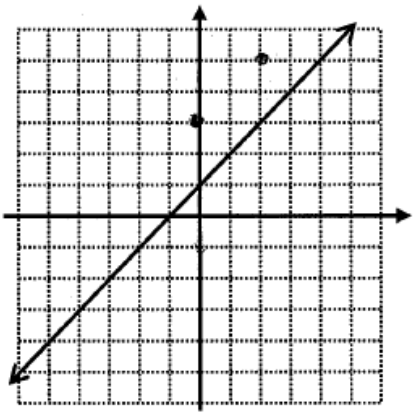
new:
$m = \frac{1}{3}$
$b = 2(2) = 4$



Equation: $y = \frac{1}{3}x + 2$

New Equation: $y = \frac{1}{3}x + 4$

~~5.~~ D(0,-1), 2



Equation: $y = x + 1$

New Equation: $y = x + 3$

6. The line $y = 2x - 4$ is dilated by a scale factor of $\frac{3}{2}$ and centered at the origin. Which equation represents the image of the line after the dilation?

(1) $y = 2x - 4$
(2) $y = 2x - 6$
(3) $y = 3x - 4$
(4) $y = 3x - 6$

$b = -4\left(\frac{3}{2}\right) = -6$

7. The equation of line h is $2x + y = 1$. Line m is the image of line h after a dilation of scale factor 4 with respect to the origin. What is the equation of the line m?

$2x + y = 1$
$y = -2x + 1$

$y = -2x + 4$

8. The line with the equation $\frac{1}{5}y = x$ is dilated by a scale factor of 3 centered at the origin. What is the equation of the new line?

$y = 5x$

6. The line $y = 2x - 4$ is dilated by a scale factor of $\frac{3}{2}$ and centered at the origin. Which equation represents the image of the line after the dilation?

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$y = -2x + 1$

$y = -2x + 4$

8. The line with the equation $\frac{1}{5}y = x$ is dilated by a scale factor of 3 centered at the origin. What is the equation of the new line?

$y = 5x$

9. Write the equation of the perpendicular bisector of the line that passes through the points (5, -8) and (-6, -2).

① $\left(\frac{5+-6}{2}, \frac{-8+-2}{2}\right)$

$(-1/2, -10/2) = (-1/2, -5)$

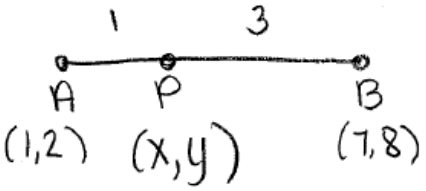
② $m = \frac{-8+(+2)}{5+(+6)} = \frac{-6}{11}$

③ $y+5 = -\frac{11}{6}\left(x+\frac{1}{2}\right)$

10. Which of the following transformations will NOT produce a line that is parallel to the line y = 4x - 7?

a) $T_{3,-2}$ b) $D_{0,4}$ (c) $R_{O,90}$ d) $r_{y=4x}$

11. Given the points A(1, 2) and B(7, 8), find the coordinates of point P on directed line segment AB that partitions AB in the ratio 1/3.



x: $\frac{1}{3} = \frac{x-1}{7-x}$

$7-x = 3x-3$

$+3+x \quad +x \quad +3$

$\frac{10}{4} = \frac{4x}{4}$ $x = \frac{5}{2}$

y: $\frac{1}{3} = \frac{y-2}{8-y}$

$8-y = 3y-6$

$+6+y \quad +y \quad +6$

$\frac{14}{4} = \frac{4y}{4}$ $y = \frac{7}{2}$

Review is long!

You may skip #'s 3,5,8,13,14,24,27,30

These are good practice but if time is an issue you can skip them. . .

Unit 10 Test Review

Midpoint Formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Slope Formula: $m = \frac{y_2-y_1}{x_2-x_1}$

Distance Formula: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Point-Slope Form: $y-y_1=m(x-x_1)$

Slope-Intercept Form: $y=mx+b$

Find the slope of $\overline{AB}$ and $\overline{CD}$ and determine if the segments are parallel, perpendicular or neither

1. A (-3, -1) B (-4, 2) | C (5,5) D (6, 2)

$x_1\ y_1\quad x_2\ y_2$

$m = \frac{2-1}{-4-3} = \frac{3}{-7} = -3$

$x_1\ y_1\quad x_2\ y_2$

$m = \frac{2-5}{6-5} = \frac{-3}{1} = -3$

Slope of $\overline{AB}$ -3

Slope of $\overline{CD}$ -3

Segments are parallel

2. A (8, 12) B (5, 4) | C (2, 1) D (5, 9)

$m = \frac{4-12}{5-8} = \frac{-8}{-3}$

$m = \frac{8}{3}$

$\frac{9-1}{5-2} = \frac{8}{3}$

Slope of $\overline{AB}$ 8/3

Slope of $\overline{CD}$ 8/3

Segments are parallel

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Find the distance between each pair of points in questions 3 to 6.

3. (10, -3) (3, 2)

$d = \sqrt{(3-10)^2+(2--3)^2}$

$\sqrt{(-7)^2+5^2}$

$\sqrt{49+25}$

$\sqrt{74}$

4. (-12, 3) (-3, 15)

$d = \sqrt{(-3--12)^2+(15-3)^2}$

$\sqrt{9^2+12^2}$

$\sqrt{81+144}$

$\sqrt{225}$

15

5. (-5, -7) (21, -8)

$d = \sqrt{(-8-7)^2+(21--5)^2}$

$\sqrt{(-1)^2+(26)^2}$

$\sqrt{1+676}$

$\sqrt{677}$

6. (-12, -7) (-6, -9)

$d = \sqrt{(-9-7)^2+(-6--12)^2}$

$\sqrt{(-2)^2+(6)^2}$

$\sqrt{4+36}$

$\sqrt{40}$

$2\sqrt{10}$

$2\sqrt{10}$

Find the length of $\overline{AB}$ and $\overline{CD}$ and determine if the segments are congruent or not congruent, in questions 7 and 8.

7. A (-3, 6) B (3, 2) C (3, 6) D (-3, -4)

AB = $\sqrt{52}$

CD = $\sqrt{136}$

Congruent? Not $\cong$

$AB=\sqrt{(3-(-3))^2+(2-6)^2}$

$\sqrt{6^2+(-4)^2}$

$\sqrt{36+16}$

$\sqrt{52}$

$CD=\sqrt{(-3-3)^2+(-4-6)^2}$

$\sqrt{(-6)^2+(-10)^2}$

$\sqrt{36+100}$

$\sqrt{136}$

8. A (0, 0) B (3, 0) C (0, -3) D (3, -3)

AB = 3

CD = 3

Congruent? are $\cong$

$AB=\sqrt{(3-0)^2+(0-0)^2}$

$\sqrt{3^2+0^2}$

$\sqrt{9}$

3

$CD=\sqrt{(3-0)^2+(-3-(-3))^2}$

$\sqrt{3^2+0^2}$

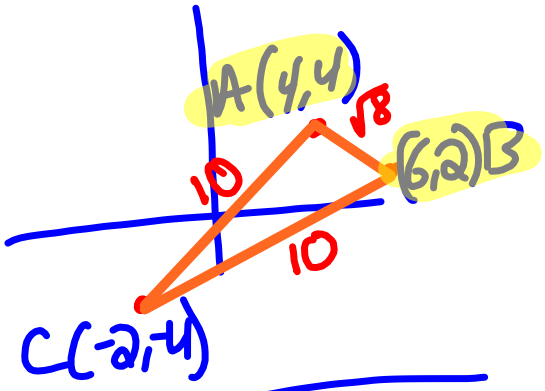
$\sqrt{9}$

3

In question 9, determine if triangle ABC is scalene, isosceles or equilateral, using the distance formula. Show work for credit.

9. A (4,4) B (6, 2) C (-2, -4)

Isosceles !



$AB = \sqrt{(6-4)^2+(2-4)^2}$

$\sqrt{2^2+(-2)^2}$

$\sqrt{4+4}$

$\sqrt{8}$

$AC = \sqrt{(4-(-2))^2+(4-(-4))^2}$

$\sqrt{6^2+8^2}$

$\sqrt{36+64}$

$\sqrt{100}$

10

$BC = \sqrt{(6-(-2))^2+(2-(-4))^2}$

$\sqrt{8^2+6^2}$

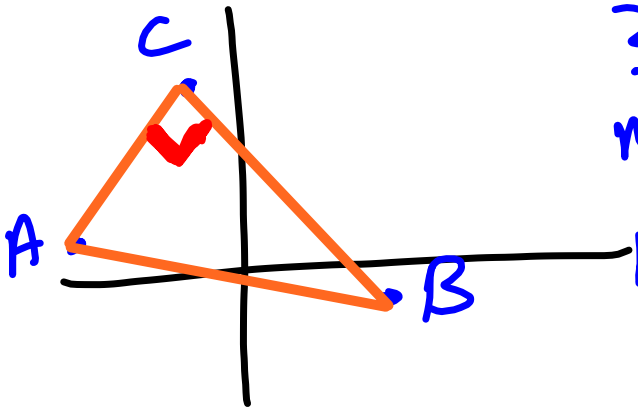
$\sqrt{64+36}$

$\sqrt{100}$

10

In question 10, determine if triangle ABC is a right triangle. Hint: Use slope to determine if two of the sides are perpendicular.

16. A (-7, 1) B(4, -2) C (-3, 5)



Rt $\triangle$

2 sides $\perp$

$m_{AB} = \frac{-2-1}{4--7} = \frac{-3}{11}$

$m_{BC} = \frac{5--2}{-3-4} = \frac{7}{-7} = -1$

$m_{AC} = \frac{5-1}{-3--7} = \frac{4}{4} = 1$

1 & $\perp$ are negative reciprocals

Rt $\triangle$ ☺ ☹ C

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Find the midpoint between the two points in questions 11 to 14. Use the midpoint formula. Be sure to answer as an ordered pair in simplest terms.

11. (-5, -3) (25, 13) (10, 5)

$x_1\ y_1\ \ x_2\ y_2$

$\left(\frac{-5+25}{2}, \frac{-3+13}{2}\right)$

$\left(\frac{20}{2}, \frac{10}{2}\right)$

$(10, 5)$

12. (35, -2) (31, 18) (33, 8)

$x_1\ y_1\ \ x_2\ y_2$

$\left(\frac{35+31}{2}, \frac{-2+18}{2}\right)$

$\left(\frac{66}{2}, \frac{16}{2}\right) = (33, 8)$

13. (26, -18) (-14, -14) (6, -16)

$\left(\frac{26+-14}{2}, \frac{-18+-14}{2}\right)$

$\left(\frac{12}{2}, -\frac{32}{2}\right) = (6, -16)$

14. (-16, 29) (-24, -27) (-20, 1)

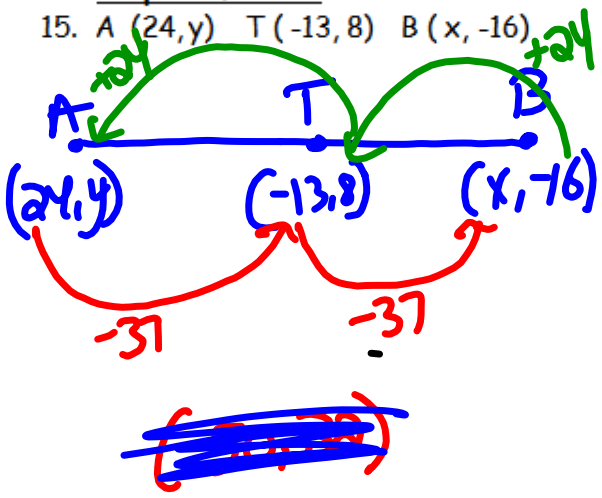
$\left(\frac{-16+-24}{2}, \frac{29+-27}{2}\right)$

$\left(\frac{-40}{2}, \frac{2}{2}\right)$

$(-20, 1)$

Find x and y in questions 15 and 16. A and B are endpoints and T is the midpoint. Use the midpoint formula.

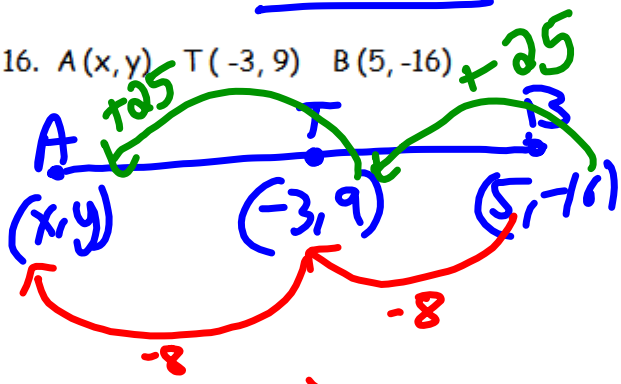
15. A (24, y) T (-13, 8) B (x, -16)



$x = -50$

$y = 32$

16. A (x, y) T (-3, 9) B (5, -16)



$(-11, 34)$

$y-y_1=m(x-x_1)$

17. What is the equation of a line through the point (1, 3) that has a slope of -2?

x, y_1 m

$y-3=-2(x-1)$

18. What is the equation of a line that passes through the coordinates (-1, 2) and (7,6)?

$y-y_1=m(x-x_1)$

$m=\frac{6-2}{7--1}=\frac{4}{8}=\frac{1}{2}$

$y-2=\frac{1}{2}(x--1)$

$y-2=\frac{1}{2}(x+1)$

19. Write the equation of a line that is parallel to y = 2x + 3 and passes through the point (6,2).

$y - y_1 = m(x - x_1)$ $m = 2$ x_1, y_1

$y - 2 = 2(x - 6)$

20. Find the equation of a line parallel to y = 4x + 12 that passes through the point (1,9).

$m = 4$ x, y_1

$y - 9 = 4(x - 1)$

21. Write the equation of a line that is perpendicular to y = 1/2x - 6 that passes through the point (6,4).

x_1, y_1

$y - 4 = -2(x - 6)$

$m = \frac{1}{2}$

$m_\perp = -\frac{2}{1}$

22. Write the equation of a line that is perpendicular to y = -3/2x - 3 that passes through the point (6,7).

x_1, y_1

$y - 7 = \frac{2}{3}(x - 6)$

$m = -\frac{3}{2}$

$m_\perp = \frac{2}{3}$

23. Write the equation of a line that is perpendicular to y = -2x + 4 that passes through the point (8,8).

x, y_1

$y - 8 = \frac{1}{2}(x - 8)$

$m = -\frac{2}{1}$ $m_\perp = \frac{1}{2}$

$2x + 3y = 4$

24. Write the equation of the perpendicular bisector that goes through the line segment with endpoints of A(2,1) and B(6, -3).

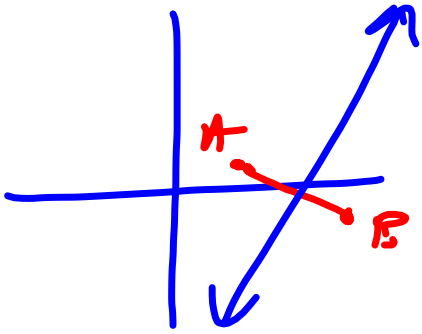
$m_{AB} = \frac{-3-1}{6-2} = \frac{-4}{4} = -\frac{1}{1}$

$m_\perp = +\frac{1}{1}m$

$\left(\frac{2+6}{2}, \frac{1+-3}{2}\right)$ Midpt:

$\left(\frac{8}{2}, \frac{-2}{2}\right) = (4, -1)$

x, y_1



$y - -1 = 1(x - 4)$

$y + 1 = 1(x - 4)$

25. Write the equation of the perpendicular bisector that goes through the line segment with endpoints of A(-1, -2) and B (-2, -8).

$m = \frac{-8--2}{-2--1} = \frac{-6}{-1} = \frac{6}{1}$ $m_\perp = -\frac{1}{6}$

Midpt: $\left(\frac{-1+-2}{2}, \frac{-2+-8}{2}\right)$

$\left(-\frac{3}{2}, \frac{-10}{2}\right)$ $\left(-\frac{3}{2}, -5\right)$

x_1 y_1

$y--5 = -\frac{1}{6}\left(x-\frac{-3}{2}\right)$

$y+5 = -\frac{1}{6}(x+3/2)$

26. Write the equation of the perpendicular bisector that goes through the line segment with endpoints of A(-1, 1) and B (7, 5)

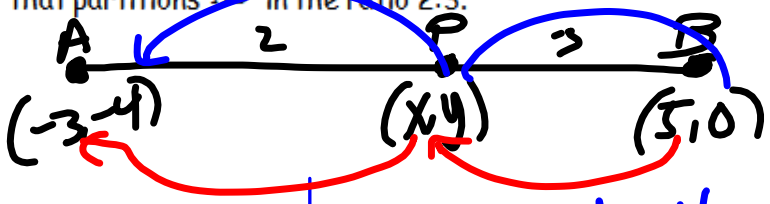
$m = \frac{5-1}{7--1} = \frac{4}{8} = \frac{1}{2}$ $m_\perp = -\frac{2}{1}$

Midpt: $\left(\frac{-1+7}{2}, \frac{1+5}{2}\right)$

$\left(\frac{6}{2}, \frac{6}{2}\right)$ $(3, 3)$

$y-3 = -2(x-3)$

27. Given the points A(-3, -4) and B(5, 0), find the coordinates of the point P on directed line segment $\overline{AB}$ that partitions $\overline{AB}$ in the ratio 2:3.



$\frac{x-(-3)}{5-x} = \frac{2}{3}$

$2(5-x) = 3(x+3)$

$10-2x = 3x+9$

$+2x \quad +2x$

$10 = 5x+9$

$-9 \quad -9$

$1 = 5x$

$x = \frac{1}{5}$

$\frac{y-(-4)}{0-y} = \frac{2}{3}$

$3(y+4) = 2(0-y)$

$3y+12 = -2y$

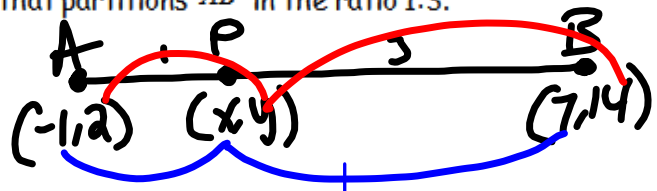
$-3y \quad -3y$

$\frac{12}{-5} = \frac{-5y}{-5}$

$y = -\frac{12}{5}$

$\left(\frac{1}{5}, -\frac{12}{5}\right)$

28. Given the points A(-1, 2) and B(7, 14), find the coordinates of the point P on directed line segment $\overline{AB}$ that partitions $\overline{AB}$ in the ratio 1:3.



$\frac{x-(-1)}{7-x} = \frac{1}{3}$

$3(x+1) = 7-x$

$3x+3 = 7-x$

$+x \quad +x$

$4x+3=7$

$-3 \quad -3$

$4x=4$

$x=1$

$\frac{y-2}{14-y} = \frac{1}{3}$

$3(y-2) = 14-y$

$3y-6 = 14-y$

$+y \quad +y$

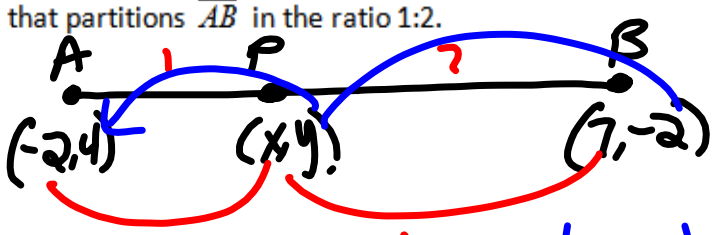
$4y-6=14$

$+6 \quad +6$

$\frac{4y}{4} = \frac{20}{4}$ $y=5$

(1, 5)

29. Given the points A(-2, 4) and B(7, -2), find the coordinates of the point P on directed line segment $\overline{AB}$ that partitions $\overline{AB}$ in the ratio 1:2.



x: $\frac{x-(-2)}{7-x} = \frac{1}{2}$

$2(x+2) = 7-x$

$2x+4 = 7-x$

$+x \quad +x$

$3x+4 = 7$

$-4 \quad -4$

$\frac{3x}{3} = \frac{3}{3}$

$x = 1$

y: $\frac{y-4}{-2-y} = \frac{1}{2}$

$2(y-4) = -2-y$

$2y-8 = -2-y$

$+y \quad +y$

$3y-8 = -2$

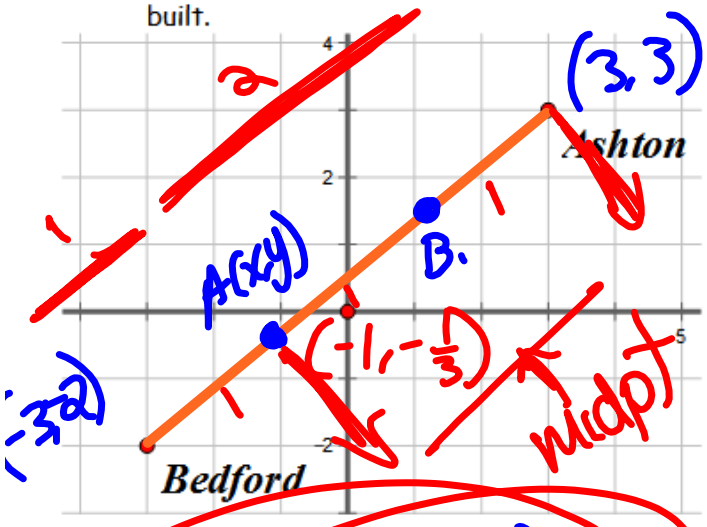
$+8 \quad +8$

$\frac{3y}{3} = \frac{6}{3}$

$y = 2$

$(1,2)$

30. The map shows a straight highway between two tows. Highway planners want to build two new rest stops between the towns so that the two rest stops divide the highway into three equal parts. Find the coordinates of the points at which the rest stops should be built.



x: $\frac{x--3}{3-x} = \frac{1}{2}$

$2(x+3) = 3-x$

$2x+6 = 3-x$

$3x+6 = 3$

$3x = -3$

$x = -1$

y: $\frac{y--2}{3-y} = \frac{1}{2}$

$2(y+2) = 3-y$

$2y+4 = 3-y$

$3y+4 = 3$

$3y = -1$

$y = -\frac{1}{3}$

1st Stop $\left(-1, -\frac{1}{3}\right)$

Midpt: $\frac{-1+3}{2}, \frac{-\frac{1}{3}+3}{2}$

$\left(\frac{2}{2}, \frac{-\frac{1}{3}+\frac{9}{3}}{2}\right)$

$1, \frac{\frac{8}{3}}{2}$

2nd $\left(1, \frac{4}{3}\right)$

31. The line with the equation $2y = 4x + 6$ is dilated by a scale factor of 2 centered at the origin. What is the equation of the new line?

$\frac{2y}{2} = \frac{4x}{2} + \frac{6}{2}$

slope same

$y = 2x + 3$ $\times 2$

$y = 2x + 3(2)$

$y = 2x + 6$

32. The equation of line m is 2x + y = 1. Line h is the image of line m after a dilation of a scale factor of 3 with respect to the origin. What is the equation of line h?

$2x + y = 1$

$-2x \quad -2x$

$y = -2x + 1$

$m = -2$

$b = 1$

$y = -2x + 1(3)$

$y = -2x + 3$