Homework Answers:

#7, 9, ~~14~~, 19, 20, 21

7. Sample spaces.

a) S = {1, 5, 10, 25} All outcomes are not equally likely. The size and weight of the coins may make some coins more likely to fall out than others.

b) S = {6, 11, 15, 26, 30, 35} All outcomes are not equally likely. The size and weight of the coins may make some coins more likely to fall out than others.

c) S = {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG} All outcomes are equally likely.

d) S = { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10} All outcomes are not equally likely. A string of 3 heads is much more likely to occur than a string of 10 heads in a row.

9. Ping pong.

We assume that there is exactly one ball with each number and that the mixing/escape process makes all the balls equally likely to be chosen. We also assume that each ball is exactly the same size, shape, and weight as all the others.

~~14. Cold streak.~~

~~There is no such thing as being "due for a hit". This statement is based on the so-called law of averages, which is a mistaken belief that probability will compensate in the short term for odd occurrences in the past. The batter's chance for a hit does not change based on recent successes or failures.~~

19. Relay team.

a) There are ${}_7C_4 = 35$ different groups of 4 swimmers each.

b) There are ${}_7P_4 = 840$ ordered groups of 4 swimmers each that the coach could create.

20. Experiment.

There are ${}_{12}C_6 = 924$ different groups of 6 plants that the researcher could choose from the 12 plants.

21. Customer satisfaction.

The store could call ${}_{21}C_4 = 5985$ different groups of customers.

What have we learned?

- Probability is based on long-run frequencies of occurrence.
- The Law of Large Numbers says these frequencies settle down to a value called the probability in the *very* long run.
  - Watch out for misinterpreting the LLN.
  - Don't fall victim to the short-run false reasoning called the "Law of Averages."

What have we learned? (cont.)

- We've learned the importance of checking the Equally Likely Condition before modeling probabilities by using:
  - sample spaces
  - the Fundamental Counting Principle
  - permutations
  - combinations
- Using probability as evidence can tell us if an observed outcome is unusual (statistically significant)

What have we learned?

- There are some basic rules for combining probabilities of outcomes to find probabilities of more complex events. We have the:
  - Probability Assignment Rule
  - Complement Rule $P(\text{No Rain}) = 1 - P(\text{Rain})$
  - Addition Rule for disjoint events
  - Multiplication Rule for independent events

What Can Go Wrong?

- Don’t mistake the Law of Large Numbers for the so-called “Law of Averages.”
- Don’t think that random events are always equally likely.
- Don’t mix up permutations and combinations.
- Don’t think a “personal probability” is mathematically valid.

What Can Go Wrong?

- Beware of probabilities that don't add up to 1.
  - To be a legitimate probability distribution, the sum of the probabilities for all possible outcomes must total 1.
- Don't add probabilities of events if they're not disjoint.
  - Events must be disjoint to use the Addition Rule.

What Can Go Wrong? (cont.)

- Don’t multiply probabilities of events if they’re not independent.
  - The multiplication of probabilities of events that are not independent is one of the most common errors people make in dealing with probabilities.
- Don’t confuse disjoint and independent—disjoint events *can’t* be independent.

Statistics Chapter 12: Review A

No homework

Statistics Chapter 12: Review A – KEY

1. Give a definition and example in your own words for each of these concepts.

   a. Law of Large Numbers

   After repeated trials over a very, very long time, the probability of an event will tend to draw closer to the actual probability. For example, to observe that the true probability of getting heads with the flip of a coin, I would need to flip the coin for many, many times before it would become extremely close to 0.5.

   b. The non-existent law of averages

   "due for ___"

   In the short run, events will work together so that they even out. For example, if you're playing roulette, and say red 23 hasn't come up in a while, you should bet on red 23 because it's due to come up. If it hasn't appeared in a long time, it needs to occur sooner rather than later. This of course is not true.

   c. Fundamental Counting Principle (for and × & or +)

   If event A can occur in m ways and event B can occur in n ways, then A OR B can occur in $m + n$ ways. Also, A AND B can occur in mn ways. Let event A be draw a jack from a standard deck and event B be roll an even number on a six-sided die. A or B can occur in 4 + 3 = 7 ways while A and B can occur in $4 \bullet 3 = 12$ ways.

   d. Permutation "Pres, position, Place

   "Arrangements"

   A permutation is when we group items in a particular way and care about the order. For example, the number of possible winners in a class election for president, vice president, secretary, and treasurer is a permutation.

   e. Combination "choose"

   group

   A combination is when we group items in a particular way but don't care about the order. For example, how many ways to group toppings on a pizza.

2. A game has two possible outcomes—win or lose. Explain when the chance of winning will not be 50%.

   The chance of winning will not be 50% when it is not a game of chance. That is, when it is a game of skill, the chance of winning is not 50%, it depends on the skill of each player.

3. There are 5 different burgers and 8 different milkshakes on a menu.

   a. How many ways can you order one burger and one milkshake?

   $5 \bullet 8 = 40$ or ${}_5C_1 \times {}_8C_1$

   5×8

   b. How many ways can you order one burger or one milkshake?

   $5 + 8 = 13$ or ${}_5C_1 + {}_8C_1$

   $5 + 8$

4. Twenty different videos will be shipped via several boxes. Three videos will be randomly selected and assigned to box A. How many different sets of videos are possible for box A?

$_{20}C_3 = 1140$ different sets of videos are possible for box A.

5. There are 12 people on a basketball team, and the coach needs to choose 5 to put into a game.

a. How many different possible ways can the coach choose a team of 5 players?

$_{12}C_5 = 792$ ways the coach can choose a team of 5.

b. If the coach chooses the 5 players at random, what is the probability that the team captain is chosen?

1 captain and 4 other

$_1C_1 \times {_{11}C_4}$

$$P(\text{Captain}) = \frac{\text{\# ways to choose the captain and 4 others}}{\text{total \# of possible teams}} = \frac{_1C_1 \bullet {_{11}C_4}}{_{12}C_5} = \frac{330}{792} \approx 0.4167$$

6. In a game, each player receives 5 cards from a deck of 52 different cards. How many different 5-card hands are possible in this game?

$_{52}C_5 = 2{,}598{,}960$ possible 5-card hands.

7. A volleyball squad has twelve players.

arrange P

a. How many ways can the players line up to greet the opposing team?

$12! = 479{,}001{,}600$ or $_{12}P_{12}$

b. There are 4 seniors on the team. What is the probability that a team of 6 players chosen randomly has all the seniors on it? 4S and 2 other

$$P(\text{All Seniors}) = \frac{\text{\# ways to choose the seniors and 2 others}}{\text{total \# of possible teams}} = \frac{_4C_4 \bullet {_8C_2}}{_{12}C_6} = \frac{28}{924} \approx 0.0303$$

8. How many license plates are possible using two letters followed by three digits, followed by one letter, as in JK233K?

$\underline{26} \bullet \underline{26} \bullet \underline{10} \bullet \underline{10} \bullet \underline{10} \bullet \underline{26} = 17{,}576{,}000$ possible license plates.

9. What is the probability that a randomly generated license plate code described in the question above starts and ends with A,E,U, or Y?

$$\frac{4 \bullet 26 \bullet 10 \bullet 10 \bullet 10 \bullet 4}{17{,}576{,}000} = \frac{416{,}000}{17{,}576{,}000} \approx 0.0237$$