Determine the interval where the given function is concaved upwards, concaved downwards, and the coordinates of all inflection points.

1) $f(x) = x^3 - 12x + 3$

$f'(x) = 3x^2 - 12$

$f''(x) = 6x = 0$

$-$ | $+$ $f''(x)$
0

concave up: $(0, \infty)$
down $(-\infty, 0)$
inflection $(0, 3)$

2) $f(x) = \frac{x}{x^2 - 1}$

$u = x \quad v = x^2 - 1$
$u' = 1 \quad v' = 2x$

$f'(x) = \frac{x^2 - 1 - 2x^2}{(x^2-1)^2} = \frac{-x^2 - 1}{x^2 - 1}$

$u = -x^2 - 1 \quad v = x^2 - 1$
$u' = -2x \quad v' = 2x$

$\frac{-\cancel{2x^3} + 2x + (+\cancel{2x^3} + 2x)}{(x^2 - 1)} = \frac{4x}{x^2 - 1}$

$f''(x) = \frac{4x}{x^2 - 1}$

n=0	d=0
$4x = 0$	$x^2 - 1 = 0$
$x = 0$	$x = \pm 1$

$-$ ⓪ $+$ ⓪ $-$ ⓪ $+$ $f''(x)$
$-1 \quad 0 \quad 1$

Concave up: $(-1, 0) \cup (1, \infty)$
" " down: $(-\infty, -1) \cup (0, 1)$
inflection: $(\cancel{-1,}\ )$ undef
$(0, 0)$
$(1, \cancel{\ \ })$ undef

3) For what values of x is the graph of $f(x) = x^3 + 6x^2 + 9x + 3$ decreasing?

A) $x \le -3$ or $x \ge -1$, only
B) $x \ge -3$, only
C) $x \le -1$, only
D) $-3 \le x \le -1$, only

$f'(x) = 3x^2 + 12x + 9$
$3x^2 + 12x + 9 = 0$
$x^2 + 4x + 3 = 0$
$(x+3)(x+1) = 0$

$+ \ \odot \ - \ \odot \ +$ $f'(x)$
$-3 \quad -1$

4) For what value(s) of x does $f(x) = 8x^4 - 4x^2$ have a relative minimum?

A) $x = 0$, only
B) $x = 0$ and $x = \frac{1}{2}$
C) $x = \frac{1}{2}$, only
D) $x = -\frac{1}{2}$ and $x = \frac{1}{2}$

$f'(x) = 32x^3 - 8x$
$8x(4x^2 - 1) = 0$

$- \ \odot \ + \ \odot \ - \ \odot \ +$ $f'(x)$
$-\frac{1}{2} \quad 0 \quad \frac{1}{2}$

G (fx)

5) The absolute maximum value of $f(x) = 4x^3 - 3x^4 + 4$ on the closed interval $[-2,3]$ occurs at $x =$

A) 1
B) -2
C) 2
D) 0

$f'(x) = 12x^2 - 12x^3$
$12x^2(1 - x) = 0$

$f(-2) = -75$ $f(3) = -131$
$f(0)$ 4
$f(1) = 5$

6) Find the coordinates of all relative extreme points for the given function below. Identify each as a relative maximum or a relative minimum.

$f(x) = 2x^3 - 9x^2 + 12x + 1$

$f'(x) = 6x^2 - 18x + 12 = 0$
$x^2 - 3x + 2 = 0$
$(x-2)(x-1) = 0$
$x = 2 \quad x = 1$

$+ \ \odot \ - \ \odot \ +$ $f'(x)$
$1 \quad 2$

rel max $(1, 6)$
rel min $(2, 5)$

7) Find the absolute maximum and the absolute minimum values of the given function on the indicated closed interval. State where these values occur.

$f(x) = \frac{x}{2} - \sin x,\ [0,\pi]$

$f'(x) = \frac{1}{2} - \cos x = 0$

$\cos x = \frac{1}{2} \quad \frac{\pi}{3}, \frac{5\pi}{3}$

$f(0) = 0 \quad f(\pi) = \frac{\pi}{2}$

$f\,\pi/3 \quad \frac{\pi}{6} - \frac{\sqrt{3}}{2}$

abs min value is 0

abs max value is $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

8) Find the coordinates of all critical points for the given function below. Classify each as a relative or absolute maximum, relative or absolute minimum, or not an extreme point.

$f(x) = 2x^3 - 3x$

$f'(x) = 6x^2 - 3 = 0$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{2}/2$

$+ \quad -\frac{\sqrt{3}}{2} \quad - \quad \frac{\sqrt{3}}{2} \quad +$

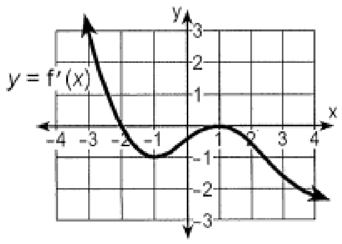
abs max $(-1, 1)$

abs min $(-\frac{\sqrt{3}}{2}, \sqrt{2}) \to (+\frac{\sqrt{3}}{2}, -\sqrt{2})$

9) The graph of the derivative of function f is shown below.

$+ \quad -2 \quad - \quad 1 \quad -$

rel max



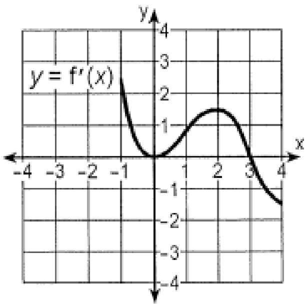
At what value of x does function f have a relative maximum?

A) 0 B) -1 C) 1 D) -2

10) The diagram below shows the graph of the derivative of function f on the open interval -1 < x < 4.

\+ 0 + 0 −

0 3



At what value(s) of x will function f have a relative maximum?

A) $x = 2$, only B) $x = 3$, only C) $x = -1$ and $x = 2$ D) $x = 0$, only

11) For the given equation:
   (a) Determine the intervals on which the given function is increasing and on which it is decreasing.
   (b) State the coordinates of all stationary points.

$f(x) = 3x^5 - 20x^3$ $f'(x) = 15x^4 - 60x^2$

$\underset{-2\qquad 0 \qquad 2}{\overline{+\ 0\ -\ 0\ +\ 0\ -}}$ $f'(x)$

inc $(-\infty,-2)\cup(2,\infty)$ $15x^2(x^2-4)=0$

dec $(-2,2)$ $(-2,64)$

rel max

rel min $(2,-64)$

12) Various values for the derivative, $f'(x)$, of a differentiable function f are shown in the table below.

x	1	2	3	4	5	6
$f'(x)$	8	4	0	-4	-8	-12

If $f'(x)$ always decreases, then which one of the following statements must be true?

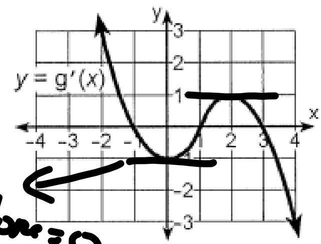
A) $f(x)$ is concaved upward for all x.
B) The graph of $f(x)$ is symmetric with the line $x = 3$.
C) $f(x)$ has a relative maximum at $x = 3$.
D) $f(x)$ changes concavity at $x = 3$.

$\underset{3}{\overline{+\ 0\ -}}$

13) The graph of the derivative of function g is shown below.

$- \; 0 \; + \; 0 \; -$

$0 \quad 2$



Slope = 0
Change in Slopes

At what value(s) of x does function g have an inflection point?

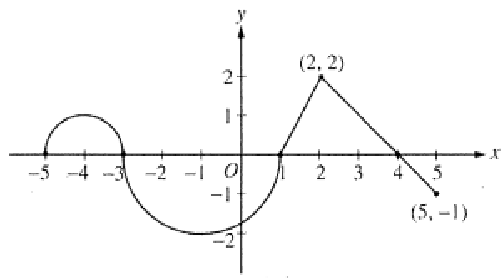
A) $x = 0$ and $x = 2$, only
B) $x = 1$, only
C) $x = 1$ and $x = 3$, only
D) $x = -1$, $x = 1$, and $x = 3$, only

Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.



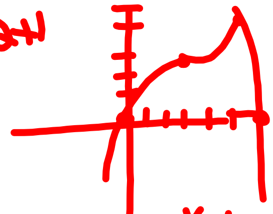
Graph of f'

a) $-3, 4$
b) $-4, -1, 2$

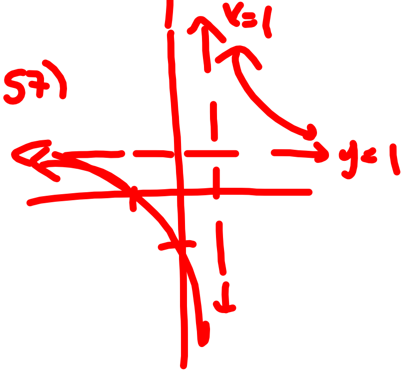
c) $(-5, -3) \cup (-1, 2)$

243

241



57)



1) A: Neither B) abs max c) neither d) neither
e) rel max f) rel min g) neither

2) a) abs min b) rel max c) N D) rel min
e) rel max f) rel min g) neither

20) $(0, \frac{5}{3})$ abs min
$(5, 5)$ abs max

24) $(2, -16)$ abs min
$(4, 16)$ abs max

22) $(0, 0)$ abs min
$(3, 0)$ abs min
$(\frac{3}{2}, \frac{9}{4})$ abs max

33) $(0, 1)$ abs max
$(\frac{1}{6}, -\frac{\sqrt{3}}{2})$ abs min