Find the inverse of : $f(x) = \sqrt{x-2}$

D $[2, \infty)$
R: $[0, \infty)$

$y = \sqrt{x-2}$
Inv $x = \sqrt{y-2}$
$x^2 = y-2$
$x^2+2 = f^{-1}(x)$ D: $[0, \infty)$
R: $[2, \infty)$

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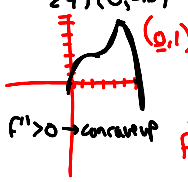
11) 0
23) $(\frac{\pi}{2}, \frac{\pi}{2})$ $(\frac{3\pi}{2}, \frac{3\pi}{2})$
24) $(0, -16)$



$(0, 1)$

$f'' > 0 \rightarrow$ concave up

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9) c 10) b 11) a 12) d 13) 1-1
14) 1-1 15) No 16) No 17) 1-1
24) No inverse 25 No inverse 26) monotonic inverse 32) $f^{-1}(x) = \sqrt[3]{x+1}$ D $(-\infty, \infty)$ R $(-\infty, \infty)$
33 43)

1	2	3/4
0	1	2/4

49) monotonic dec inverse

$f(x) = (x+2)^2(x-4)$
$(x^2+4x+4)(x-4)$
$x^3 - 4x^2 + 4x^2 - 16x + 4x - 16$
$6x = 0$
$x = 0$

$- \; 0 \; +$
0

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The derivative of an inverse formula: $\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$

Ex1) Find $f^{-1\prime}(a)$ if $f(x) = x^3 + x + 1$ and $a = 1$

$y = x^3 + x + 1$
$x = y^3 + y + 1$

$1 = x^3 + x + 1$
$-1 \qquad -1$
$0 = x^3 + x$
$0 = x(x^2+1)$
$0 = x$

$f^{-1\prime}(x) = \frac{1}{f'(0)}$

$f'(x) = 3x^2 + 1$
$f'(0) = 3(0)^2 + 1 = 1$

$f^{-1\prime}(x) = \frac{1}{1} = 1$

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Ex2) Let $y = x^3 + x - 2$ and let g be the inverse function. Evaluate g '(0).

$0 = x^3 + x - 2$ q p

p: $\pm 1 \pm 2$
q: ± 1
p/q $= \pm 1 \; \pm 2$

1	1	0	1	-2
	↓	1	1	2
	1	1	2	0

$x^2 + x + 2 = 0$

$\frac{1}{f'(1)}$

$y'(x) = 3x^2 + 1$
$y'(1) = 3(1)^2 + 1 = 4$

$= \frac{1}{4}$

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Ex3) If f(3)=8 and f '(3)=5 what do we know about the inverse graph of f.

point on orig	pt inverse
$(3, 8)$	$(8, 3)$
slope = 5 at x=3	slope = 1/5 at x=8

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Ex4) If $f(x) = \sqrt{x-4}$ and g(x) = inverse of f(x) find g'(1)

$y = \sqrt{x-4}$ $[4, \infty)$ D $[0, \infty)$ R
Inverse: $x = \sqrt{y-4}$
$x^2 = y-4$
$x^2+4 = f^{-1}(x)$ $[0, \infty)$ D $[4, \infty)$ R

$g(x) = x^2 + 4$
$g'(x) = 2x$
$g'(1) = 2(1)$
$= 2$

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Let $f(x)=\frac{1}{4}x^3+x-1$

a) What is the value of $f^{-1}(x)$ when x =3?

$3=\frac{1}{4}x^3+x-1$ $f^{-1}(x)=2$

b) What is the value of $(f^{-1})'(x)$ when x=3

$\frac{1}{f'(f^{-1}(x))} \quad \frac{1}{f'(2)}$ $=\frac{1}{4}$

$f'(x)=\frac{3}{4}x^2+1$

$\frac{3}{4}(2)^2+1=4$

f

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dcr

find the domain and ranges of f and f^{-1} show that the slopes of the graphs of f and f^{-1} are reciprocals at the given point

a) $f(x)=x^3$ $(\frac{1}{2},\frac{1}{8})$ d:$(-\infty,\infty)$ r:$(-\infty,\infty)$

$f^{-1}(x)=\sqrt[3]{x}$ $(\frac{1}{8},\frac{1}{2})$ $(-\infty,\infty)$ $(-\infty,\infty)$ $x^{\frac{1}{3}}$

$f'(x)$	$f^{-1}{}'(x)$
$f'(x)=3x^2$	$f^{-1}{}'(x)=\frac{1}{3}x^{-2/3}$
$f'(\frac{1}{2})=3(\frac{1}{2})^2$	$f^{-1}{}'(x)=\frac{1}{3\sqrt[3]{x^2}}$
$\frac{3}{4}$	$f^{-1}{}'(\frac{1}{8})=\frac{1}{3\sqrt[3]{(\frac{1}{8})^2}}$

$\frac{1}{\frac{3}{4}}$ $4/3$

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b) $f(x)=\sqrt{x-4}$ (5,1) d:$[4,\infty)$ r:$[0,\infty)$

$f^{-1}(x)=x^2+4$ (1,5) d:$[0,\infty)$ r:$[4,\infty)$

$f'(x)$	$f^{-1}{}'(x)$
$f'(x)=\frac{1}{2\sqrt{x-4}}$	$f^{-1}{}'(x)=2x$
$f'(5)=\frac{1}{2\sqrt{1}}$	$f^{-1}{}'(1)=2$
$\frac{1}{2}$	reciprocal slopes

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Use the function f(x)=2x-4 and g(x)=3x-1 find each below:

a) $g^{-1}\circ f^{-1}$

$g^{-1}(x)=$ $y=3x-1$ inv: $x=3y-1$ $\frac{x+1}{3}=\frac{3y}{3}$ $g^{-1}(x)=\frac{x+1}{3}$

$f^{-1}(x)$ $y=2x-4$ inv: $x=2y-4$ $\frac{x+4}{2}=f^{-1}(x)$

$f^{-1}(x)=\frac{x+4}{2}$

$g^{-1}(\frac{x+4}{2})=\frac{\frac{x+4}{2}+\frac{1}{1}\cdot\frac{2}{2}}{\frac{3}{1}\cdot 2}$

$\frac{x+4+2}{6}=\frac{x+6}{6}$

b) $(g\circ f)^{-1}$

$f(2x-4)=2x-4$

$g(2x-4)=3(2x-4)-1$

$6x-13=y$ inv: $6y-13=x$

$6y=x+13$

$y=\frac{x+13}{6}$

$(g\circ f)^{-1}=\frac{x+13}{6}$

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Cross vertical / horizontal line test

$f(x)=x^2+8x+1$

$f'(x)=2x+8=0$ $-\ \ominus\ +$ -4 rel min

Not 1-1 change in signs

$f(x)=x^3$

$f'(x)=3x^2$ $+\ \ominus\ +$ 0 1-1

monotonic inc.

monotonic increase → + 1st der

decrease − 1st der

ex1) $f(x)=2-x-x^3$

$f'(x)=-1-3x^2=0$ $-\frac{1}{3}=\frac{3x^2}{3}$ no critical values 1-1 monotonic dec

ex2) $f(x)=x^3-x+1$

$f'(x)=3x^2-1=0$ $+\ \ominus\ -\ \ominus\ +$ $f'(x)$

$x^2=\frac{1}{3}$ $-\frac{\sqrt{3}}{3}$ $\frac{\sqrt{3}}{3}$

$x=\pm\frac{\sqrt{3}}{3}$ Not 1-1 not monotonic

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Inverses

$f(x)=2x+5$ $g(x)=\frac{x-5}{2}$ inverses?

$f(g(x))=x$ and $g(f(x))=x$

$f(\frac{x-5}{2})=2(\frac{x-5}{2})+5=x$ ✓ yes

$g(f(x))=g(2x+5)=\frac{2x+5-5}{2}=x$

find inverse of $f(x)=x^3+2$ d: $(-\infty,\infty)$ r: $(-\infty,\infty)$

$y=x^3+2$

$x=y^3+2$

$\sqrt[3]{x-2}=\sqrt[3]{y^3}$

$\sqrt[3]{x-2}=y$

$f^{-1}(x)=\sqrt[3]{x-2}$ D:$(-\infty,\infty)$ r:$(-\infty,\infty)$

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$f(x) = \frac{2x-4}{x-1}$

d: $x \neq 1$ $(-\infty, 1) \cup (1, \infty)$

r: H.A: $x-1 \overline{)2x-4}$ quotient 2, $(-\infty, 2) \cup (2, \infty)$

$y = 2$

Cross:

$\frac{2}{1} = \frac{2x-4}{x-1}$

$\cancel{2x} - 2 = \cancel{2x} - 4$

NO

Inverse

$y = \frac{2x-4}{x-1}$

$\frac{x}{1} = \frac{2y-4}{y-1}$

$xy - x = 2y - 4$
$\quad +x \qquad +x$

$xy - 2y = x - 4$

$\frac{y(x-2)}{(x-2)} = \frac{x-4}{x-2}$

$f^{-1}(x) = \frac{x-4}{x-2}$

d: $(-\infty, 2) \cup (2, \infty)$

r: $(-\infty, 1) \cup (1, \infty)$

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$f(1) = 5$ $f(3) = 7$ $f(8) = -10$

$f^{-1}(7) = 3$ $f^{-1}(5) = 1$ $f^{-1}(-10) = 8$

$f(x) = \sqrt{x-4}$ find $g(x)$ and $g'(1)$

$x \geq 4$

d: $[4, \infty)$

r: $[0, \infty)$

$y = \sqrt{x-4}$

$x = \sqrt{y-4}$

$x^2 = y - 4$

$x^2 + 4 = y$

$f^{-1}(x) = x^2 + 4$

d: $[0, \infty)$

r: $[4, \infty)$

$g(x) = x^2 + 4$

$g'(x) = 2x$

$g'(1) = 2(1) = 2$

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