NEWTONS METHOD → approximates zeros of a function

Algorithm. Let f be a differentiable function. Choose a point x_1 near a root of f. Define recursively

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

start value; f(x_n) → original function; f'(x_n) → derivative

If the point x_1 is chosen sufficiently close to the root then the x_n's are successively better approximations of the root.

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Steps: Make an initial estimate x_1 that is close to root

Determine a new approximation using the formula above

Go until the sequence converges

Ex1) Calculate three iterations of Newton's method to approximate a zero of $f(x) = x^2 - 2$ start at x = 1

$f'(x) = 2x$

$x - \frac{f(x)}{f'(x)}$

1st: $1 - \frac{f(1)}{f'(1)} \Rightarrow 1 - \frac{-1}{2} = \frac{3}{2}$ (1, 1.5)

2nd: $\frac{3}{2} - \frac{f(3/2)}{f'(3/2)}$ $\quad \frac{3}{2} - \left(\frac{(\frac{3}{2})^2 - 2}{2(3/2)}\right)$ (1.5, $\frac{17}{12}$)

3rd: $17/12 - \frac{f(17/12)}{f'(17/12)} = 1.414215686$ ($\frac{17}{12}$, $\frac{577}{408}$ 1.414215686)

actual: calc: 1.41423562

Solve

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

.3562

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Ex2) Use Newtons method to approximate the zeros of $f(x) = 2x^3 + x^2 - x + 1$ first 2

zero

Let x = −1.2

actual calc →

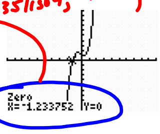
$f'(x) = 6x^2 + 2x - 1$

1st $-1.2 - \frac{f(-1.2)}{f'(-1.2)} =$

2nd = −1.233754022

(−1.2, −1.23511504)

(−1.23511504, −1.233754022)



Calculator use for Newton method

y_1 → enter original

y_2 → derivative

y_3 → y_1 / y_2

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Ex3) The function $f(x) = x^{\frac{1}{3}}$ is not differentiable at x=0. Show that Newton's method fails to converge using $x_1 = .1$

$f'(x) = \frac{1}{3}x^{-2/3}$

$\frac{1}{3\sqrt[3]{0}}$ undef

$.1 - \frac{f(.1)}{f'(.1)} = -.199994443$

$-.19994443 - \frac{f(-.19994443)}{f'(-.199443)} = .399861107$

Never converges

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Apply Newtons method to approximate the x-values of the indicated point(s) of intersection of the two graphs. Continue until two successive approximations differ by less than .001

use $x_1 = .7$

$f(x) = 2x + 1$

$g(x) = \sqrt{x+4}$

$2x + 1 = \sqrt{x+4}$

$2x + 1 - \sqrt{x+4} = 0$

$h(x) = 2x + 1 - \sqrt{x+4}$ → y_1

$(x+4)^{\frac{1}{2}}$

$h'(x) = 2 - \frac{1}{2}(x+4)^{-\frac{1}{2}}$

$h'(x) = 2 - \frac{1}{2\sqrt{x+4}}$

1st = $.7 - \frac{h(.7)}{h'(.7)} = .568850478 5$ → (.7, .5688504785)

3rd: (.5687293045, .5687293044) → (.5688504785, .5687293045)

Solution

Ans to question

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Consider the function $f(x) = x^3 - 3x^2 + 3$

a) Use Newton methods with $x_1 = 1$ as an initial guess.

b) Repeat part b using $x_1 = 1/4$ as the initial guess and observe that the result is different

a)

```
1→X
                 1
X-Y3→X
       1.333333333
       1.347222222
       1.347296353
       1.347296355
```

b)

```
X-Y3→X
       2.404761905
       2.556193434
        2.53272158
       2.532089341
       2.532088886
       2.532088886
```

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