Name: Answer Key Date:

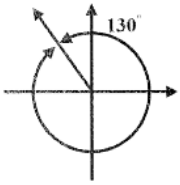
UNIT #7 – UNIT CIRCLE TRIG
COMMON CORE ALGEBRA II

Part I Questions

1. Which of the following angles is coterminal with an angle of 130°, assuming both angles are drawn in the standard position?

(1) 230° (3) 430°

(2) -230° (4) -310°



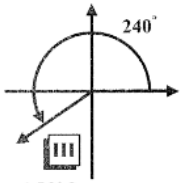
$360^\circ - 130^\circ = 230^\circ$ clockwise $\Rightarrow -230^\circ$

(2)

2. If drawn in the standard position, which of the following angles terminates in the third quadrant?

(1) 120° (3) -210°

(2) -60° (4) 240°

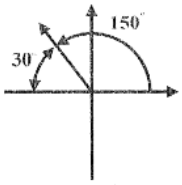


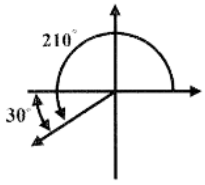
(4)

3. Which of the following has the same reference angle as 150°?

(1) 210° (3) 120°

(2) 300° (4) 70°





(1)

4. The radian angle $\frac{3\pi}{4}$ is equivalent to

(1) 67.5° (3) 270°

(2) 135° (4) 325°

$$\frac{3\pi}{4} \times \frac{180^\circ}{\pi} = \frac{3}{4}(180) = 135^\circ$$

(2)

5. The angle 240° can be written equivalently as which of the following in the radian system?

(1) $\frac{7\pi}{6}$ (3) $\frac{3\pi}{2}$

(2) $\frac{5\pi}{4}$ (4) $\frac{4\pi}{3}$

$$240^\circ \times \frac{\pi}{180^\circ} = \frac{240\pi}{180} = \frac{24\pi}{18} = \frac{4\pi}{3}$$

(4)

6. A point lies on the unit circle whose x-coordinate is $\frac{1}{4}$. If the point lies in the fourth quadrant, then which of the following is its y-coordinate?

(1) $\frac{3}{4}$ (3) $-\frac{\sqrt{7}}{4}$

(2) $-\frac{\sqrt{15}}{4}$ (4) $\frac{\sqrt{11}}{2}$

$$x^2 + y^2 = 1$$
$$\left(\frac{1}{4}\right)^2 + y^2 = 1$$
$$y^2 + \frac{1}{16} = 1$$
$$y^2 = \frac{15}{16}$$
$$y = \pm\sqrt{\frac{15}{16}} = \pm\frac{\sqrt{15}}{\sqrt{16}} = \pm\frac{\sqrt{15}}{4}$$

We are told that the point lies in the fourth quadrant. In the fourth quadrant, the y-coordinates are negative. Thus, the correct answer is:

$$y = -\frac{\sqrt{15}}{4}$$

(2)

7. Which of the following could *not* be the value of the cosine of an angle?

(1) $-\frac{4}{5}$ (3) $\frac{\sqrt{11}}{4}$

(2) $\frac{7}{3}$ (4) $-\frac{\sqrt{3}}{2}$

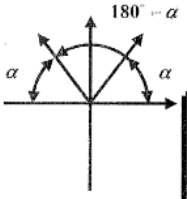
All values of cosine (and sine) must be between -1 and 1 (inclusive). Hence, because $\frac{7}{3}$ is larger than 1, it cannot be a cosine value.

(2)

8. If α is an angle such that $0^\circ < \alpha < 90^\circ$ and $\sin\alpha = 0.56$ then which of the following is the value of $\sin(180 - \alpha)$?

(1) 0.56 (3) 0.83

(2) –0.56 (4) –0.83



From this diagram, we can tell that the angle $180 - \alpha$ will have the same sine value as the angle α because they would be at the same y-value on the unit circle.

(1)

9. If $f(x) = 10\sin(2x) + 8$ then $f\left(\frac{\pi}{4}\right) = ?$

(1) $4\sqrt{2}$ (3) 18

(2) 8 (4) $28\sqrt{3}$

$$f\left(\frac{\pi}{4}\right) = 10\sin\left(2\cdot\frac{\pi}{4}\right) + 8$$
$$= 10\sin\left(\frac{\pi}{2}\right) + 8$$
$$= 10(1) + 8$$
$$= 18$$

(3)

10. If an angle has a positive cosine but a negative sine then it must terminate in which of the following quadrants?

(1) I (3) III

(2) II (4) IV

$\cos A > 0 \Rightarrow$ I or IV
and
$\sin A < 0 \Rightarrow$ III or IV

Therefore, the angle must be located in Quadrant IV to make both of these conditions true.

(4)

11. The terminal ray of an angle drawn in standard position passes through the point $(.508,.862)$ on the unit circle. Which of the following is closest to the tangent of this angle?

(1) .685 (3) 1.697

(2) 1.291 (4) 2.883

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\text{y-coordinate}}{\text{x-coordinate}} = \frac{.862}{.508} = 1.69685... \approx 1.697$

(3)

12. If α is an angle drawn in the standard position with its terminal ray landing in the fourth quadrant and $\csc(\alpha) = -5$, then which of the following is the exact value of $\cos(\alpha)$?

(1) $-\frac{1}{5}$ (3) $\frac{\sqrt{24}}{5}$

(2) $-\frac{24}{25}$ (4) $\frac{\sqrt{6}}{2}$

$\csc\alpha = -5 \Rightarrow \sin\alpha = -\frac{1}{5}$

$\cos^2\alpha + \sin^2\alpha = 1$

$\cos^2\alpha + \left(-\frac{1}{5}\right)^2 = 1$

$\cos^2\alpha + \frac{1}{25} = 1$

$\cos^2\alpha = \frac{24}{25}$

$\cos\alpha = \pm\sqrt{\frac{24}{25}} = \pm\frac{\sqrt{24}}{5}$

in the fourth quadrant cosine is positive so...

$\cos\alpha = \frac{\sqrt{24}}{5}$

(3)

13. For the angle θ it's known that $\cot(\theta) < 0$ and $\sin(\theta) > 0$. In which quadrant does the terminal ray of θ lie?

(1) I (3) III

(2) II (4) IV

$\cot\theta < 0 \Rightarrow$ II or IV

and

$\sin\theta > 0 \Rightarrow$ I or II

thus must be II

(2)

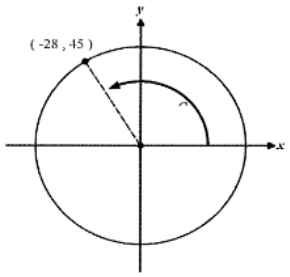
Free Response Questions

14. An angle drawn in standard position measures 10 radians. In what quadrant does its terminal ray lie? Show the reasoning that leads to your answer.

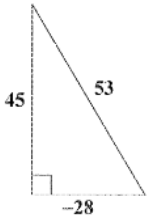
$10 \text{ radians} \times \frac{180^\circ}{\pi} = 572.96^\circ$ ⇨ Since this angle is greater than 360°, we need to subtract 360° from the angle to pass one full rotation. ⇨ $572.96^\circ - 360^\circ = 212.96^\circ$

Quadrant III

15. Given the following circle (note that it is **not** the **unit circle**) with the angle θ marked, state the values of each of the following:



(a) The radius of the circle $28^2 + 45^2 = r^2 \Rightarrow r^2 = 2809 \Rightarrow r = \sqrt{2809} = 53$



(b) $\sin\theta = \frac{45}{53}$

(c) $\cos\theta = -\frac{28}{53}$

(d) $\tan\theta = \frac{45}{-28} = -\frac{45}{28}$

(e) $\sec\theta = \frac{1}{\cos\theta} = -\frac{53}{28}$

(f) $\csc\theta = \frac{1}{\sin\theta} = \frac{53}{45}$

(g) $\cot\theta = \frac{1}{\tan\theta} = -\frac{28}{45}$

16. For an angle A it is known that $\sin A = \frac{3}{4}$ and $\cos A < 0$. Determine the value of $\tan A$. Show how you arrived at your answer.

$\cos^2 A + \sin^2 A = 1$

$\cos^2 A + \left(\frac{3}{4}\right)^2 = 1$

$\cos^2 A + \frac{9}{16} = 1$

$\cos^2 A = \frac{7}{16}$

$\cos A = -\sqrt{\frac{7}{16}} = -\frac{\sqrt{7}}{4}$

$\tan A = \frac{\sin A}{\cos A}$

$= \frac{\frac{3}{4}}{-\frac{\sqrt{7}}{4}} = \frac{3}{\cancel{4}} \cdot -\frac{\cancel{4}}{\sqrt{7}} = -\frac{3}{\sqrt{7}}$ or $-\frac{3\sqrt{7}}{7}$