Name: _Key_   Algebra 2 Homework 11-7

Period : _____

Problems from emathinstruction:

1. Evie is on a swing thinking about trigonometry (no seriously!). She realizes that her height above the ground is a periodic function of time that can be modeled using $h = 3\cos\left(\frac{\pi}{2}t\right) + 5$, where t represents time in seconds. Which of the following is the range of Evie's heights?

$$\text{min} = -3 + 5 = 2$$
$$\text{max} = 3 + 5 = 8$$
$$\boxed{2 \leq h \leq 8}$$

1) $2 \leq h \leq 8$

2. The height of a yo-yo above the ground can be well modeled using the equation $h = 1.75\cos(\pi t) + 2.25$, where h represents the height of the yo-yo in feet above the ground and t represents time in seconds since the yo-yo was first dropped from its maximum height.

(a) Determine the maximum and minimum heights that the yo-yo reaches above the ground. Show the calculations that lead to your answers

$$\text{min}: -1.75 + 2.25 = \boxed{.5 \, ft}$$
$$\text{max}: 1.75 + 2.25 = \boxed{4 \, ft}$$

(b) How much time does it take for the yo-yo to return to the maximum height for the first time?

$$Per = 2\pi/w! = 2\pi/\pi = \boxed{2 \, sec}$$

3. The possible hours of daylight in a given day is a function of the day of the year. In Poughkeepsie, New York, the minimum hours of daylight (occurring on the Winter solstice) is equal to 9 hours and the maximum hours of daylight (occurring on the Summer solstice) is equal to 15 hours. If the hours of daylight can be modeled using a sinusoidal equation, what is the equation's amplitude?
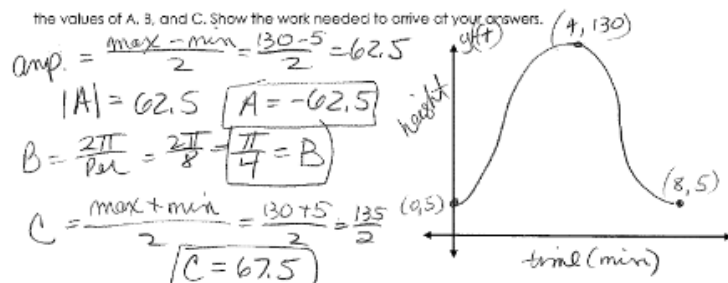
a) 6   b) 12   c) 3   d) 4

$$amp = \frac{max - min}{2} = \frac{15 - 9}{2} = \boxed{3}$$

4. A Ferris wheel is constructed such that a person gets on the wheel at its lowest point, five feet above the ground, and reaches its highest point at 130 feet above the ground. The amount of time it takes to complete one full rotation is equal to 8 minutes. A person's vertical position, y, can be modeled as a function of time in minutes since they boarded, t, by the equation $y = A\cos(Bt) + C$. Sketch a graph of a person's vertical position for one cycle and then determine the values of A, B, and C. Show the work needed to arrive at your answers.

$$\text{amp.} = \frac{max - min}{2} = \frac{130 - 5}{2} = 62.5$$

$$|A| = 62.5 \quad \boxed{A = -62.5}$$

$$B = \frac{2\pi}{Per} = \frac{2\pi}{8} = \boxed{\frac{\pi}{4} = B}$$

$$C = \frac{max + min}{2} = \frac{130 + 5}{2} = \frac{135}{2}$$

$$\boxed{C = 67.5}$$

Graph labeled: y(t), height vs time (min), points $(4, 130)$, $(8, 5)$, $(0, 5)$.

5. A ball is attached to a spring, which is stretched and then let go. The height of the ball is given by the sinusoidal equation $y = -3.5\cos\left(\frac{4\pi}{5}t\right) + 5$, where y is the height above the ground in feet and t is the number of seconds since the ball was released.

(a) At what height was the ball released at? Show the calculation that leads to your answer.

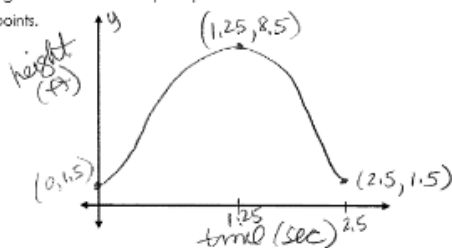$$t = 0 \qquad y = -3.5\cos(0) + 5 = -3.5 + 5 = \boxed{1.5 \text{ ft}}$$

(b) What is the maximum height the ball reaches?

$$max = 3.5 + 5 = \boxed{8.5 \text{ ft}}$$

(c) How many seconds does it take the ball to return to its original position?

$$Per = \frac{2\pi}{|\omega|} = \frac{2\pi}{4\pi/5} = \frac{2\pi}{1} \cdot \frac{5}{4\pi} = \frac{10}{4} = \boxed{2.5 \text{ sec}}$$

(d) Draw a rough sketch of one complete period of this curve below. Label maximum and minimum points.

Graph labeled: height (ft) vs time (sec), points $(1.25, 8.5)$, $(0, 1.5)$, $(2.5, 1.5)$.

In-Class Review ANSWERS:

1. 2

2. $[-2, 2]$

3. 2 in 2π

4. π

5. $\pi/4$

6. See graph

7. $[-8, 8]$

8. 7.3 inches

9. -2

10. 3

11. $f(x) = 3\cos\left(\frac{1}{2}x\right)$

12. $\pi/2$ seconds

13. $f(x) = 3\cos\left(\frac{\pi}{4}x\right) + 4$

14.
a. 15
b. -35
c. -5
d. $[-35, -5]$
e. $\frac{2\pi}{2\pi} = 1$
f. 1
g. 8 right
h. 20 down

15.
a. 10 ft.
b. 6 ft.
c. 2
d. 8 up
e. 12 hrs.
f. 1/12
g. $\pi/6$
h. $f(t) = -2\cos\left(\frac{\pi}{6}t\right) + 8$
i. 6.1 ft.

For questions 1 to 6, use the equation $f(x) = -2\sin(2x)$

1) Find the amplitude.

2) Find the range.

3) Find the frequency. $|w|/2\pi = 2$ cycles in 2π
$$\text{or } 1 \text{ in } \pi$$

4) Find the period. $2\pi/2 = \pi$
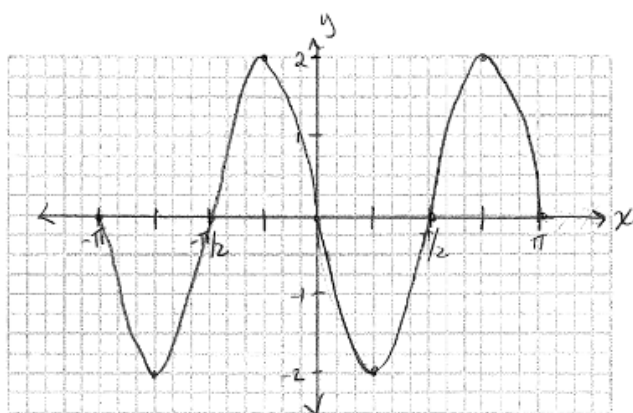
5) Find the x-scale. $\pi/4$

1) 2

2) $[-2,2]$

3) 2 in 2π

4) π

5) $\pi/4$

6) Graph the equation in the interval $-\pi \leq x \leq \pi$.

Pattern: $-2(0, 1, 0, -1, 0) = (0, -2, 0, 2, 0)$

7) What is the range for f(x) = 8 sin(x) ?

7) $[-8, 8]$

8) The motion of an object can be modeled by the equation
   f(x) =2,3 sin (πx) + 5, where x represents the number of seconds the
   object is oscillating and x is the distance, in inches, of the height of
   the object. What is the maximum height of the object?

8) 7.3

$$+2.3 \quad +5 = 7.3$$

9) If f(x) = 5 + 7 sin(x), what is the minimum value of the function?

9) -2

$$\underset{5}{\downarrow} \; \underset{-7}{\downarrow} = -2$$

10) Which graph has an amplitude of 5 and a period of π ?

10) 3

$|w| = \frac{2\pi}{\pi} = 2$

1) $y = \frac{1}{5}\cos(4x)$
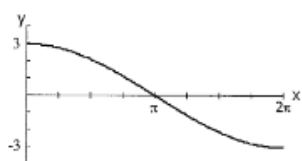
2) $y = 5\sin(4x)$

3) $y = 5\sin(2x)$

4) $y = 5\cos\left(\frac{1}{4}x\right)$

11) Write the equation of the function represented by the graph below:

11) $f(x) = 3\cos\left(\frac{1}{2}x\right)$

$amp = 3, \quad a = 3$

$|w| = \frac{1}{2} / 2\pi$

12) The voltage E of an alternating current electrical circuit can be represented by the function E(t) = 660 cos (4t), where E is measured in volts and t is measured in seconds. How long does it take the alternating current to complete one full cycle?

12) $\frac{\pi}{2}$ sec.

$per = \frac{2\pi}{4} = \frac{\pi}{2}$ sec.

13) The following graph represents the vertical translation of a basic trigonometric curve. Write an equation of the translated graph. Show all work.
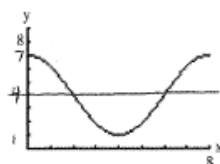
amp = 3　　　　cosine

Per = 8

$|\omega| = 2\pi/8 = \pi/4$

$K = 4$

$$f(x) = 3\cos\left(\frac{\pi}{4}x\right) + 4$$

14) Describe the graph for $f(x) = 15\cos(2\pi(x-8)) - 20$.

a) amplitude　=　__15__

b) minimum　=　-15-20 = -35

c) maximum　=　15-20 = -5

d) range　=　[-35, -5]

e) frequency　=　2π in 2π = 1　　$|\omega| = 2\pi$

f) period　=　__1__

g) phase shift　=　8 to Rt.

h) vertical shift =　Down 20

15) The tide at a boat dock can be modelled by the following graph.   Let t= the number of hours past noon and y is the height of the tide in feet.

Height

(in feet)

Time past noon (hours)
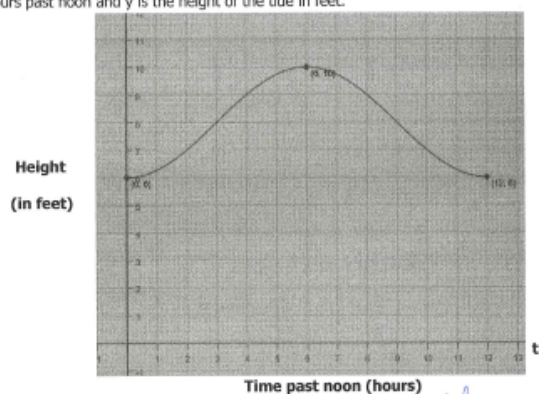
a. What is the maximum water depth? 10 ft.

b. What is the minimum water depth? 6 ft.

c. What is the amplitude? 2

d. What is the vertical shift (midline)? 8 (up)

e. What is the period? 12 hrs.

f. What is the frequency? $\frac{1}{12}$

g. What is the $|\omega|$? $\frac{2\pi}{12} = \frac{\pi}{6}$

h. What is the function of the height of the tide, f(t), as shown in the diagram above?
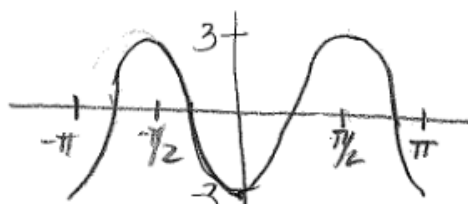
$$f(t) = -2\cos\left(\frac{\pi}{6}t\right) + 8$$

i. What would be the height of the tide, to the nearest tenth, at 11:30 PM?

$$t = 11.5$$

$$f(11.5) = -2\cos\left(\frac{\pi}{6}(11.5)\right) + 8 = 6.1 \text{ ft.}$$

(Radian mode)

Unit 11 Extra Practice

1) Graph the equation **f(x)=-3cos(2x) from** $-\pi \le x \le \pi$. Be sure to state the amplitude, frequency and period as part of your work.

$amp = 3$ $|w| = 2$ $per = \frac{2\pi}{2} = \pi$ $xsc\ell = \frac{\pi}{4}$

$freq = \frac{2}{2\pi}$

2) What is the range for $y = -3\sin(x)$?

$$[-3, 3]$$

3) The motion of a spring can be modeled by the function f(x) =2.4 cos (πx) - 7, where x represents the number of seconds the spring is oscillating and y is the distance, in inches, of the height of the spring. What are the maximum and minimum heights of the spring?

$Max: 2.4 - 7 = -4.6$ $Min = -2.4 - 7 = -9.4$

4) If f(x) = -6 + 4 sin(2x), what is the maximum value of the function?

$-6 + 4 = -2$

5) Give an example of a function that has an amplitude of 4 and a period of π ?

$f(x) = 4\sin(2x)$ $|w| = \frac{2\pi}{\pi} = 2$

6) The voltage E of an alternating current electrical circuit can be represented by the function
    E(t) = 220 cos (4πt), where E is measured in volts and t is measured in seconds. How long
    does it take the alternating current to complete one full cycle?

$$|w| = 4\pi \qquad Per = length \ of \ 1 \ cycle = \frac{2\pi}{|w|} = \frac{2\pi}{4\pi}$$
$$= \boxed{\frac{1}{2} \ sec}$$

7) Describe the graph for $y = 15\cos(\pi(x+3)) - 10$.

a) amplitude   = $\underline{\quad 15 \quad}$

b) frequency   = $\underline{\pi \ in \ 2\pi = 1/2}$

c) period   = $\underline{\frac{2\pi}{\pi} = 2}$

d) phase shift   = $\underline{3 \ left}$

e) vertical shift = $\underline{Down \ 10}$

f) minimum = $\underline{-15 - 10 = -25}$

g) maximum = $\underline{15 - 10 = 5}$

h) range = $\underline{[-25, 5]}$

8) A pet store clerk noticed that the population in the gerbil habitat varied sinusoidally with respect to time, in days. He carefully collected data and graphed his resulting equation. From the graph, determine the amplitude, period, horizontal shift and vertical shift. Write the equation of the graph. (regentsprep.org)
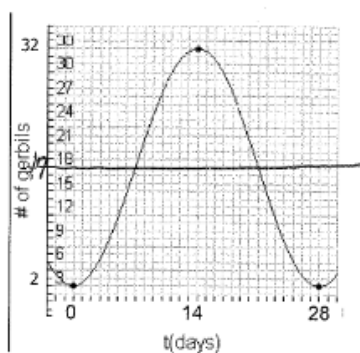
a. What is the amplitude? $\frac{32-2}{2} = 15$ ___15___ $a = -15$

b. What is the period?   ___28 days___

c. What is the frequency?   ___1 cycle in 28 days___

d. What is $|\omega|$? $2\pi/28 = \pi/14$ ___$\pi/14$___

e. What is the vertical shift (midline)? $\frac{32+2}{2}$ ___17___

f. What is the horizontal shift?   ___0___

g. What is the equation of the gerbil population, P(t), as shown in the diagram above as a cosine function?

$$P(t) = -15\cos\left(\tfrac{\pi}{14}t\right) + 17$$

h. What would be the population when t = 45 days?

$$P(45) = -15\cos\left(\tfrac{\pi}{14}(45)\right) + 17 = 28.7 \sim 29 \text{ days}$$

calc in radians!