Average Rate of Change

HW 9-1

1) a. -5
b. 2
c. $-\frac{1}{4}$
d. See explanation

2) a. 2
b. 6
c. 10
d. See explanation

3) $g(x)$
See work

4) a. 22
b. $11 \le t \le 14$

5) See explanation

Name: ______________________________ Date: ______________

AVERAGE RATE OF CHANGE
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. For the function $g(x)$ given in the table below, calculate the average rate of change for each of the following intervals.

x	-3	-1	4	6	9
$g(x)$	8	-2	13	12	5

(a) $-3 \le x \le -1$

$= \dfrac{-2-8}{-1-(-3)} = \dfrac{-10}{2} = -5$

(b) $-1 \le x \le 6$

$= \dfrac{12--2}{6--1} = \dfrac{14}{7} = 2$

(c) $-3 \le x \le 9$

$= \dfrac{5-8}{9--3} = \dfrac{-3}{12} = -\dfrac{1}{4}$

(d) Explain how you can tell from the answers in (a) through (c) that this is **not** a table that represents a linear function.

If this was a linear function then the average rate of change would have been the same for each of these intervals.

2. Consider the simple quadratic function $f(x)=x^2$. Calculate the average rate of change of this function over the following intervals:

(a) $0 \le x \le 2$

$= \dfrac{f(2)-f(0)}{2-0}$

$\dfrac{4-0}{2} = 2$

(b) $2 \le x \le 4$

$\dfrac{f(4)-f(2)}{4-2}$

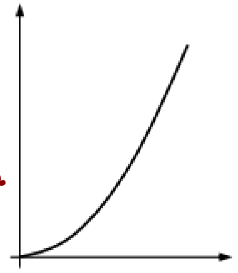
$\dfrac{16-4}{2} = 6$

(c) $4 \le x \le 6$

$\dfrac{f(6)-f(4)}{6-4}$

$\dfrac{36-16}{2} = 10$

(d) Clearly the average rate of change is getting larger at x gets larger. How is this reflected in the graph of f shown sketched to the right?



As x gets larger the y-values are increasing in larger intervals.

or

The graph is getting steeper as we move from left to right.

3. Which has a greater average rate of change over the interval $-2 \le x \le 4$, the function $g(x) = 16x - 3$ or the function $f(x) = 2x^2$? Provide justification for your answer.

$g(x)$

$$= \frac{g(4) - g(-2)}{4 - -2}$$

$$= \frac{61 - -35}{6} = \frac{96}{6} = 16$$

$f(x)$

$$= \frac{f(4) - f(-2)}{4 - -2}$$

$$= \frac{32 - 8}{6} = \frac{24}{6} = 4$$

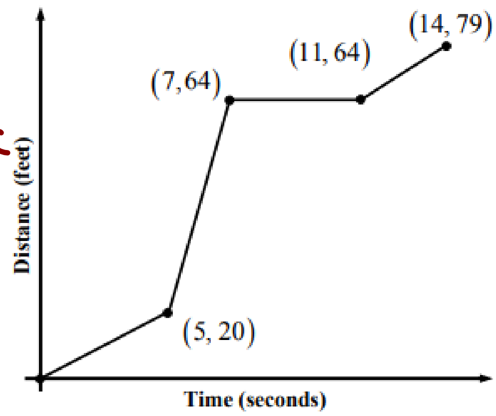
$g(x)$ has a greater rate of change.

APPLICATIONS

4. An object travels such that its distance, *d*, away from its starting point is shown as a function of time, *t*, in seconds, in the graph below.

(a) What is the average rate of change of *d* over the interval $5 \le t \le 7$? Include proper units in your answer.

$$= \frac{64 - 20}{7 - 5} = \frac{44}{2} = 22 \text{ ft/sec}$$



(b) The average rate of change of distance over time (what you found in part (a)) is known as the **average speed** of an object. Is the average speed of this object greater on the interval $0 \le t \le 5$ or $11 \le t \le 14$? Justify.

$0 \le t \le 5$ $\qquad$ $11 \le t \le 14$

$$\frac{20 - 0}{5 - 0} = 4 \frac{\text{ft}}{\text{sec}} \qquad \frac{79 - 64}{14 - 11} = 5 \frac{\text{ft}}{\text{sec}}$$

The average speed is slightly greater on the interval $11 \le t \le 14$

REASONING

5. What makes the average rate of change of a linear function different from that of any other function? What is the special name that we give to the average rate of change of a linear function?

The average rate of change is a constant for linear functions and is not dependent on the interval over which it is calculated. We call this average rate of change the slope.

Rules of Exponents

Rules of Exponents

Unit 9 Day 2

xxxxx · xxx

	Rule:	Examples:
Product Rule ➔	$x^a \bullet x^b = x^{a+b}$	a. $x^5 \bullet x^3 = x^8$
		b. $2^x \bullet 2^3 = 2^{x+3}$
Quotient Rule ➔	$\frac{x^a}{x^b} = x^{a-b}$	a. $\frac{x^7}{x^2} = x^5$
		* b. $\frac{4^x}{4^3} = 4^{x-3}$
Power Rule ➔	$(x^a)^b = x^{ab}$	a. $(x^3)^4 = x^{12}$
		b. $(3^2)^3 = 3^6$

$\frac{xxxxx\cancel{xx}}{\cancel{xx}}$

Power of a Product ➔ $(xy)^a = x^a y^a$

a. $(ab)^6 = a^6 b^6$

b. $2^3 \cdot 3^3 = 6^3$

$(2 \cdot 3)^3$

Power of a Quotient ➔ $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

a. $\left(\frac{x}{y}\right)^7 = \frac{x^7}{y^7}$

b. $\left(\frac{x^3}{4}\right)^2 = \frac{x^6}{4^2} = \frac{x^6}{16}$

Zero Exponent ➔ $x^0 = 1$

a. $3\cancel{x^0}^{1} = 3(1) = 3$

b. $(3x)^0 = 1$

c. $-(3x)^0 = -1$

✵ 0^0 → undefined

Simplify each expression:

1. $3a^2b^3c^4 \cdot 5ab^2c^6$

$15a^3b^5c^{10}$

2. $\dfrac{35x^4y^7z^{10}}{7xy^5z^{10}} = 5x^3y^2$

3. $3(2x^3y^2)^3$

$3(\cancel{2}^{8}x^9y^6)$

$24x^9y^6$

4. $-2(-2x^3y)^2$

$-2(4x^6y^2) = -8x^6y^2$

5. $\dfrac{x^{3b}}{x^b} = x^{3b-b} = x^{2b}$

6. $y^{a+1} \cdot y^{a-1} = y^{a+1+a-1} = y^{2a}$

7. $\frac{5x^3y^7}{4x^2y^2} = \frac{5xy^5}{4}$

$\frac{5}{4}xy^5$

$\cancel{\frac{5}{4}xy^5}$

9. $\left(\frac{-12a^8b^5}{6a^2b^4}\right)^2 = (-2a^6b)^2$

$= 4a^{12}b^2$

8. $\left(\frac{5x^2}{2y}\right)^3 = \frac{125x^6}{8y^3}$

10. $-\frac{4^0}{5} = -\frac{1}{5}$

<u>Do the following without a calculator. Use the rules of exponents to help you evaluate the expression.</u>

base 2^x

Express 8^3 as a power of 2.

$8^3 = 2^x$

$(2^3)^3 = 2^x$

$2^9 = 2^x$

$8^3 = 2^9$

Divide 4^{15} by 2^{10}.

$\frac{4^{15}}{2^{10}} = \frac{(2^2)^{15}}{2^{10}} = \frac{2^{30}}{2^{10}} = 2^{20}$

of a product

Using the power rule evaluate 16 times 9.

$$4^2 \cdot 3^2 = (4 \cdot 3)^2 = 12^2 = 144$$

Using the power rule multiply 25 times 9.

$$5^2 \cdot 3^2 = (5 \cdot 3)^2 = 15^2 = 225$$

Apply the properties of exponents to verify that each statement is an identity.

$$\frac{2^{n+1}}{3^n} = 2\left(\frac{2}{3}\right)^n$$

$$\frac{2^{n+1}}{3^n} = \frac{2^1 \cdot 2^n}{3^n}$$

$$\frac{2^{n+1}}{3^n} = \frac{2^{n+1}}{3^n} \checkmark$$

$$\frac{3^{n+1}}{3^n} - \frac{3^n}{3^n} = 2(3^n)$$

$$3^n(3^1 - 1) = 2(3^n)$$

$$3^n(2) = 2(3^n)$$

$$2(3^n) = 2(3^n) \checkmark$$