HW 9-4

1. 4
2. 32
3. $\frac{1}{27}$
4. -8
5. 25
6. $125x^9$
7. $\frac{1}{27x^3}$
8. $\sqrt[5]{5^3}$ or $\sqrt[5]{125}$
9. $5\sqrt[5]{x^3}$
10. $\sqrt[5]{(x+6)}$
11. $x^{\frac{3}{4}}$
12. $7x^{\frac{4}{7}}$
13. $(x+9)^{\frac{5}{4}}$
14. $7^{\frac{5}{9}}$
15. $10^{\frac{35}{36}}$
16. $5^{\frac{5}{12}}$
17. $b^{\frac{2}{3}}$
18. $7^{\frac{7}{24}}$
19. $11^{\frac{2}{35}}$
20. $y^{\frac{1}{30}}$
21. $x^{\frac{1}{4}}$
22. 1
23. $\frac{3}{\sqrt{x^3}}$ or $\frac{3}{(\sqrt{x})^3}$

Name_____ Alg 2 HW9-4

Simplify. Write your answers as fractions, if necessary.

1. $64^{\frac{1}{3}} = 4$

2. $16^{\frac{5}{4}}$
$= (\sqrt[4]{16})^5$
$= 2^5 = 32$

3. $81^{-\frac{3}{4}}$
$= \frac{1}{(\sqrt[4]{81})^3}$
$= \frac{1}{3^3} = \frac{1}{27}$

4. $(-32)^{\frac{3}{5}}$
$= (\sqrt[5]{-32})^3$
$= (-2)^3$
$= -8$

5. $\left(-\frac{1}{125}\right)^{-\frac{2}{3}}$
$= (-125)^{2/3}$
$= (\sqrt[3]{-125})^2 = (-5)^2 = 25$

6. $(25x^6)^{\frac{3}{2}}$
$= (\sqrt{25x^6})^3$
$= (5x^3)^3 = 125x^9$

7. $\left(\frac{1}{9x^2}\right)^{\frac{3}{2}}$
$= (\sqrt{\frac{1}{9x^2}})^3$
$= \left(\frac{1}{3x}\right)^3 = \frac{1}{27x^3}$

State each expression as a simplified radical expression.

8. $5^{\frac{3}{5}}$
$= \sqrt[5]{5^3}$
or
$\sqrt[5]{125}$

9. $5x^{\frac{3}{5}}$
$= 5\sqrt[5]{x^3}$

10. $(x+6)^{\frac{1}{5}}$
$= \sqrt[5]{(x+6)}$

State each radical expression in exponential form.

11. $\sqrt[4]{x^3}$ $= x^{3/4}$

12. $7\sqrt[7]{x^4}$ $= 7x^{4/7}$

13. $\sqrt[4]{(x+9)^5}$ $= (x+9)^{5/4}$

$\frac{1}{}$

$\frac{1}{}$

Multiply and simplify.

14. $7^{\frac{1}{9}} \cdot 7^{\frac{4}{9}}$

$= 7^{5/9}$

15. $10^{\frac{2}{9}} \cdot 10^{\frac{3}{4}}$

$= 10^{8/36} \cdot 10^{27/36}$

$= 10^{35/36}$

16. $\sqrt[12]{5} \cdot \sqrt[3]{5}$

$= 5^{1/12} \cdot 5^{1/3}$

$= 5^{1/12} \cdot 5^{4/12}$

$= 5^{5/12}$

Divide and simplify.

17. $\frac{b^{\frac{7}{9}}}{b^{\frac{1}{9}}}$

$= b^{6/9}$

$= b^{2/3}$

18. $\frac{7^{\frac{3}{8}}}{7^{\frac{1}{12}}}$

$= \frac{7^{9/24}}{7^{2/24}}$

$= 7^{7/24}$

19. $\frac{\sqrt[5]{11}}{\sqrt[7]{11}}$

$= \frac{11^{1/5}}{11^{1/7}}$

$= 11^{7/35} = 11^{2/35}$

Simplify.

20. $\frac{\sqrt[5]{y}}{y^{\frac{1}{2}}} \cdot y^{\frac{2}{3}}$

$= \frac{y^{1/5}}{y^{1/2}} \cdot y^{2/3}$

$= y^{6/30} \cdot y^{20/30} = y^{26/30} = y^{11/30}$

21. $\frac{\sqrt[3]{x} \cdot x^{\frac{1}{6}}}{(x^2)^{\frac{1}{8}}}$

$= \frac{x^{1/3} \cdot x^{1/6}}{x^{1/4}} = \frac{x^{4/12} \cdot x^{2/12}}{x^{3/12}}$

$= \frac{x^{6/12}}{x^{3/12}} = x^{1/4}$

$\frac{1}{y^{9/30}} \cdot \frac{y^{2/3}}{1} = \frac{y^{2/3}}{y^{9/30}}$

22. If $f(x) = 3x^{-\frac{1}{4}}$, find f(81)

$= 3(81)^{-1/4} = 3\left(\frac{1}{81}\right)^{1/4} = 3\sqrt[4]{1/81} = 3 \cdot \frac{1}{3} = 1$

23. Written without fractional or negative exponents, $3x^{-3/2}$ is equal to

$3x^{-3/2} = \frac{3}{x^{3/2}} = \frac{3}{\sqrt{x^3}}$ or $\frac{3}{(\sqrt{x})^3}$

Operations with Fractional Exponents and Simplifying Radical Expressions

Unit 9 Day 5

Operations with Fractional Exponents and Simplifying Radical Expressions

Write the following expressions as a single term with a rational exponent.

1. $\sqrt[3]{x} \cdot \sqrt{x}$ $= x^{1/3} \cdot x^{1/2}$ $= x^{1/3+1/2}$ $= x^{5/6}$

2. $\sqrt[4]{2a} \cdot \sqrt[3]{2a}$ $= (2a)^{1/4} \cdot (2a)^{1/3}$ $= (2a)^{1/4+1/3}$ $= (2a)^{7/12}$

Using the rules for positive and negative exponents, simplify the following expressions.

3a. $\left(\frac{m^2}{m^{\frac{1}{3}}}\right)^{-\frac{1}{2}}$ $= \left(m^{2-1/3}\right)^{-1/2}$ $= \left(m^{5/3}\right)^{-1/2}$ $= m^{-5/6}$ or $\frac{1}{m^{5/6}}$ or $\frac{1}{\sqrt[6]{m^5}}$

b. $\left(\frac{x^2}{x^{\frac{1}{2}}}\right)^{-\frac{1}{3}}$ $= \left(x^{2-1/2}\right)^{-1/3}$ $= \left(x^{3/2}\right)^{-1/3}$ $= x^{-1/2}$ or $\frac{1}{x^{1/2}}$ or $\frac{1}{\sqrt{x}}$

4. For $x \neq 0$, which expression(s) are equivalent to one divided by the sixth root of x?

a) $\frac{\sqrt[6]{x}}{\sqrt[3]{x}} = \frac{x^{1/6}}{x^{1/3}}$ $\Rightarrow$ b) $\frac{x^{\frac{1}{6}}}{x^{\frac{1}{3}}}$ c) $x^{-\frac{1}{6}}$ $\frac{1}{\sqrt[6]{x}}$

$x^{1/6-1/3}$

$x^{-1/6} = C$

With the following two questions, you will need to think about how you can eliminate a fractional exponent so that the variable that you are solving for is to the first power.

5. Solve algebraically for n: $m^{-\frac{2}{7}} \cdot \sqrt[4]{n^5} = m^{\frac{8}{7}}$

New!

$$\frac{m^{-2/7} \cdot n^{5/4}}{m^{-2/7}} = \frac{m^{8/7}}{m^{-2/7}}$$

$$\left(n^{5/4}\right)^{4/5} = \left(m^{10/7}\right)^{4/5}$$

$$n^1 = m^{40/35}$$

$$n = m^{8/7}$$

6. Given the equal terms $\sqrt[3]{x^5}$ and $y^{\frac{5}{6}}$, determine and state y in terms of x.

Solve for y =

$$\sqrt[3]{x^5} = y^{5/6}$$

$$\left(x^{5/3}\right)^{4/5} = \left(y^{5/6}\right)^{4/5}$$

$$x^{20/15} = y^1$$

$$x^2 = y$$

When written as a radical $\sqrt[n]{b}$, the **radicand** is *b* and the **index** is *n*. The denominator of the exponent in $a^n = b$ is also the index of the radical. If the index is odd, a negative number can have a negative root. If the index is even, the radicand must be positive to have a real value.

Powers to Memorize: perfect squares, perfect cubes, etc

Base / Power	2	3	4	5
2	4	8	16	32
3	9	27	81	243
4	16	64	256	X
5	25	125	625	X

7. Simplify. Identify the index and the radicand.

a. $\sqrt[3]{-27} = -3$

I: 3

R: -27

b. $\sqrt[4]{16} = 2$

I = 4

R = 16

c. $\sqrt[3]{8y^3}$

d. $\sqrt{4x^6}$

Students try

$\sqrt[3]{64x^6} = 4x^{6/3} = 4x^2$

I = 3

R = $64x^6$

$\sqrt[4]{81y^{12}} = 3y^{12/4} = 3y^3$

Product and Quotient Rule for Radicals

Product Rule: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$, with $\sqrt[n]{a}$ and $\sqrt[n]{b}$ as real numbers

Quotient Rule: $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ with b≠0, and $\sqrt[n]{a}$ and $\sqrt[n]{b}$ as real numbers

8. Simplify using the product rule or quotient rule.

a. $\sqrt[3]{48x^3y^5}$

$\sqrt[3]{8x^3y^3} \cdot \sqrt[3]{6y^2}$

$2xy\sqrt[3]{6y^2}$

b. $\sqrt[4]{64x^{10}}$

$\sqrt[4]{16x^8} \cdot \sqrt[4]{4x^2}$

$2x^2\sqrt[4]{4x^2}$

c. $\sqrt{48x^5y^8}$

d. $\frac{\sqrt[3]{16x^8}}{\sqrt[3]{2x^2}} = \sqrt[3]{\frac{16x^8}{2x^2}} = \sqrt[3]{8x^6}$

$2x^{6/3}$

$2x^2$

e. $\frac{\sqrt{3x^2}}{\sqrt{12x}} = \sqrt{\frac{3x^2}{12x}} = \sqrt{\frac{x}{4}}$

$= \frac{\sqrt{x}}{\sqrt{4}} = \frac{\sqrt{x}}{2}$

f. $\frac{\sqrt[4]{5y^5}}{\sqrt[4]{16y}}$

$\sqrt[4]{\frac{5y^5}{16y}}$

$\sqrt[4]{\frac{5y^4}{16}}$

$\frac{\sqrt[4]{5y^4}}{\sqrt[4]{16}} = \frac{y\sqrt[4]{5}}{2}$

Students try:

$\sqrt[3]{54x^4y^9}$

$\sqrt[3]{27x^3y^9} \cdot \sqrt[3]{2x}$

$3xy^3\sqrt[3]{2x}$

$\frac{\sqrt[4]{2a^{15}}}{\sqrt[4]{32a^3}} = \sqrt[4]{\frac{a^{12}}{16}} = \frac{a^3}{2}$