Unit 10 Day 7 HW

1. $f(n) = 6(2)^{4n}$
2. (3)
3. ~~5200 years~~ 5169.53 years
4. (a) 16600 people (b) 11.3 years
5. t = 23.22 minutes
6. h = 10.3002 t = 18.6 hours

Name________________________________ Alg 2 CC
Date________________ Unit 10 Day 7 HW

1. A computer application generates a sequence of musical notes using the function $f(n) = 6(16)^n$, where n is the number of the note in the sequence and $f(n)$ is the note frequency in hertz. Write a function that will generate the same note sequence as $f(n)$ with a base of 2.

$f(n) = 6(2^4)^n$

$f(n) = 6(2)^{4n}$

2. Iridium-192 is an isotope of iridium and has a half-life of 73.83 days. If a laboratory experiment begins with 100 grams of Iridium-192, the number of grams, A, of Iridium-192 present after t days would be

$A = 100\left(\frac{1}{2}\right)^{\frac{t}{73.83}}$ $= 100\left(\left(\frac{1}{2}\right)^{1/73.83}\right)^t = 100(.990656)^t$

Which equation approximates the amount of Iridium-192 present after t days?

$\left(\frac{1}{2}\right)^{1/73.83}$

(1) $A = 100\left(\frac{73.83}{2}\right)^t$

(2) $A = 100\left(\frac{1}{147.66}\right)^t$

(3) $A = 100(0.990656)^t$

(4) $A = 100(0.116381)^t$

3. The half-life of radium is every 1690 years. If 10 grams are present now, how long will it take to the nearest hundredth of a year, to have 1.2 grams of radium?

5169.53

$$\frac{1.2}{10} = \frac{10\left(\frac{1}{2}\right)^{t/1690}}{10}$$

$$.12 = \left(\tfrac{1}{2}\right)^{t/1690}$$

$$\frac{\log(.12)}{\log(\frac{1}{2})} = \frac{\frac{t}{1690}\log\left(\frac{1}{2}\right)}{\log(\frac{1}{2})}$$

$$\frac{\log(.12)}{\log(\frac{1}{2})}\cdot 1690 = \frac{t}{1690}\cdot 1690$$

4. The current population of Little Pond, New York is 20,000. The population is decreasing, as represented by the formula $P = A(1.3)^{-0.234t}$, where P = final population, t = time in years, and A = initial population.

a. What will the population be 3 years from now? Round your answer to the nearest hundred people.

$$P = 20{,}000(1.3)^{-.234(3)}$$
$$= 16635.7$$
$$= 16600 \text{ people}$$

b. To the nearest tenth of a year, how many years will it take the population to reach half the present population?

$$\frac{1}{2} = (1.3)^{-.234t}$$

$$\frac{\log\left(\frac{1}{2}\right)}{-.234\log(1.3)} = \frac{-.234t\log(1.3)}{-.234\log(1.3)}$$

$$t = 11.29 \approx 11.3 \text{ years}$$

5. The number of bacteria in a petri dish doubles every 10 minutes. How many minutes to the nearest hundredth of a minute will it take for the bacteria to be 5 times the original amount?

$$5 = 1(2)^{t/10}$$

$$\frac{\log 5}{\log 2} = \frac{\frac{t}{10} \log 2}{\log 2}$$

$$10 \cdot \frac{\log 5}{\log 2} = \frac{t}{10} \cdot 10$$

$t = 23.22$ minutes

6. A radioactive substance has a mass of 140 g at 3 p.m. and 100 g at 8 p.m.

Write an equation in the form $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$ that models this situation, where h is the constant representing the number of hours in the half-life, A_0 is the initial mass, and A is the mass t hours after 3 p.m.

Using this equation, solve for h, to the nearest ten thousandth.

$8\text{ PM} - 3\text{ PM}$

$t = 5$

$$\frac{100}{140} = \frac{140\left(\frac{1}{2}\right)^{5/h}}{140}$$

$$\frac{100}{140} = \left(\frac{1}{2}\right)^{5/h}$$

$$\log_{\frac{1}{2}}\left(\frac{100}{140}\right) = \frac{5}{h}$$

$$\frac{\log\left(\frac{10}{14}\right)}{\log\left(\frac{1}{2}\right)} = \frac{\frac{5}{h}\cancel{\log\left(\frac{1}{2}\right)}}{\cancel{\log\left(\frac{1}{2}\right)}}$$

$$\frac{.4854268272}{1} = \frac{5}{h}$$

$$\frac{.4854268272h}{.4854268272} = \frac{5}{.4854268272}$$

$h = 10.3002$

Determine when the mass of the radioactive substance will be 40 g. Round your answer to the nearest tenth of an hour.

$$\frac{40}{140} = \frac{140\left(\frac{1}{2}\right)^{t/10.3002}}{140}$$

$$\frac{\log\left(\frac{4}{14}\right)}{\log\left(\frac{1}{2}\right)} = \frac{\frac{t}{10.3002}\log\left(\frac{1}{2}\right)}{\log\left(\frac{1}{2}\right)}$$

$$\frac{\log\left(\frac{4}{14}\right)}{\log\left(\frac{1}{2}\right)} \cdot 10.3002 = \frac{t}{10.3002} \cdot 10.3002$$

$t = 18.6$ hours

The number e.

e is a special irrational number. e stands for Euler' number. Leonhard Euler (oy-ler) was a Swiss mathematician & physicist.

$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$

$e \approx 2.718$

Evaluate the following to the nearest hundredth:

1. $e^{2.47}$ 11.82

2. $e^{3.51}$ 33.45

Graph $y = e^x$ and its inverse. For graph of e, create a table with your calculator.

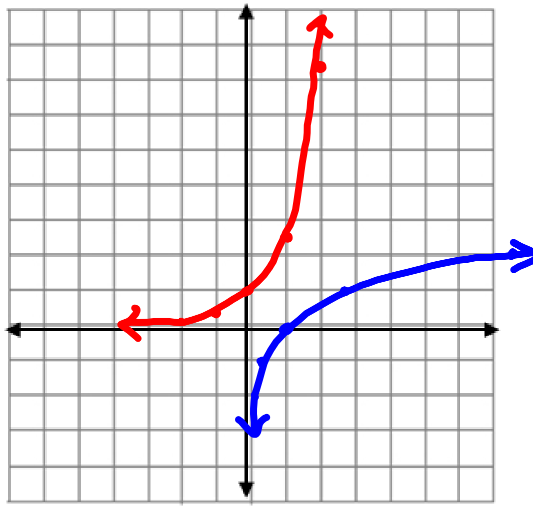
$y = e^x$

x	y
-2	.14
-1	.37
0	1
1	2.7
2	7.4

INV: $y = \log_e x$

$y = \ln x$

x	y
.14	-2
.37	-1
1	0
2.7	1
7.4	2



Because of the importance of $y = e^x$, its **inverse**, known as the **natural logarithm**, is also

THE NATURAL LOGARITHM
The inverse of $y = e^x$: $y = \ln x \quad (y = \log_e x)$

important. The natural logarithm, like all logarithms, gives an exponent as its output. In fact, it gives the power that we must raise **e** to in order to get the input.

Equation of the inverse:
$y = \ln x$

$y = e^x$ function D: $(-\infty, \infty)$

R: $(0, \infty)$

ln e = $\log_e e$ = 1

$e^1 = e$

log base e → ln

$y = \ln x$ inverse D: $(0, \infty)$

R: $(-\infty, \infty)$

Properties of logs apply to natural logs.

ln 1 = $\log_e 1 = 0$ ← $e^0 = 1$

ln e= $\log_e e = 1$

$e^{\ln x} = e^{\log_e x} = x$

$e^{\log_e x} = ?$

$\log_e ? = \log_e x$

$? = x$

Write each expression as a single natural logarithm.

1. $3 \ln 5 + 1/2 \ln x$

$\ln(5^3) + \ln(x^{1/2})$

$\ln(125\sqrt{x})$

2. $\ln 24 - \ln 6$

3. $1/3(\ln x + \ln y) - 4 \ln z$

$\frac{1}{3}\ln(xy) - 4\ln z$

$\ln\left(\frac{\sqrt[3]{xy}}{z^4}\right)$

4. $\ln a - 2 \ln b + \frac{1}{2} \ln c$

Expand each natural logarithm.

5. $\ln \frac{a^2 b^3}{\sqrt{c}}$ $c^{1/2}$

$2\ln a + 3\ln b - \frac{1}{2}\ln c$

6. $\ln (2x)^2$

$2\ln(2x)$

$2(\ln 2 + \ln x)$

$2\ln 2 + 2\ln x$

7. $\ln 49xyz$

8. $\ln \frac{\sqrt[3]{r}}{st}$

Solve. Round to the nearest hundredth where necessary.

9. $e^x = 18$

$e^x = 18$

$\log_e 18 = x$

$\ln 18 = x$

$\ln e^x = \ln 18$

$x \cancel{\ln e} = \ln 18$

$x = 2.89$

10. $e^{x/5} + 4 = 7$

$-4 \quad -4$

$e^{x/5} = 3$

$\log_e 3 = \frac{x}{5}$

$\frac{x}{5} \cancel{\ln e} = \ln 3$

$(5)\frac{x}{5} = (5)\ln 3$

$x = 5.49$

11. $25 = e^{.075t}$

$\frac{\ln 25}{.075} = \frac{.075t \cancel{\ln e}}{.075}$

$t = 42.92$

12. $\frac{10}{5} = \frac{5e^k}{5}$

$2 = e^k$

$\ln 2 = k \cancel{\ln e}$

$k = .69$

13. $7 - 2e^{x/2} = 1$

$-7 \quad -7$

$\frac{-2e^{x/2}}{-2} = \frac{-6}{-2}$

$e^{x/2} = 3$

$(2)\frac{x}{2}\cancel{\ln e} = 2\ln 3$

$x = 2.20$

14. $\ln 3x = 4$

$\log_e 3x = 4$

$\frac{e^4}{3} = \frac{3x}{3}$

$x = 18.20$

15. $2 \ln x - 1 = 7$

$\frac{2\ln x}{2} = \frac{8}{2}$

$\log_e x = 4$

$e^4 = x$

$x = 54.60$