Unit 10 Day 8 HW

1. $x = .5$
2. $x = 2.71$
3. $x = 0.92$
4. $x = 1.91$
5. $x = 17.33$
6. $x = 0.61$
7. $x = 201.71$
8. $\ln 243$
9. $\ln \dfrac{x^4}{\sqrt[3]{y}}$
10. $\ln \dfrac{x^2\sqrt{y}}{zw^3}$

    or

    $\ln \dfrac{x^2\sqrt{y}}{w^3 z}$
11. $3\ln x + \frac{1}{2}\ln y$
12. $2\ln a + 3\ln b - \ln c - \frac{1}{2}\ln d$
13. $3\ln x - 2$

Name________________________________ Alg 2 CC

Date________________________ Unit10 HW Day8

1. Using logarithms, find the value of x to the nearest tenth.

$$7^{(x+1)} = 18.6$$

$$\frac{(x+1)\log 7}{\log 7} = \frac{\log 18.6}{\log 7}$$

$x = 0.5$

$x+1 = 1.5022$
$-1 \quad -1$
$x = .5$

Use natural logarithms to solve each equation. Round your answer to the nearest hundredth.

2. $e^x = 15$

$x \cancel{\ln e}^{1} = \ln 15$

$x = 2.71$

3. $\frac{4e^x}{4} = \frac{10}{4}$

$e^x = \frac{5}{2}$

$x \cancel{\ln e}^{1} = \ln(5/2)$

$x = .92$

4. $e^{x+2} = 50$

$(x+2)\cancel{\ln e}^{1} = \ln 50$
$-2 \qquad -2$
$x = 1.91$

5. $e^{\frac{x}{5}} - 4 = 28$

$+4 \quad +4$
$e^{x/5} = 32$
$\frac{x}{5}\cancel{\ln e}^{1} = \ln 32$
$x = 5 \cdot \ln 32$
$x = 17.33$

6. 4 ln x = -2

$\frac{4\ln x}{4} = \frac{-2}{4}$
$\log_e x = -\frac{1}{2}$
$e^{-1/2} = x$
$x = .61$

7. ln 2x = 6

$\log_e 2x = 6$
$\frac{e^6}{2} = \frac{2x}{2}$
$x = 201.71$

Write each expression as a single natural logarithm.

8. 3 ln 3 + ln 9

$\ln 3^3 \cdot 9$
$\ln 243$

9. 4 ln x - 1/3 ln y

$\ln \frac{x^4}{y^{1/3}} = \ln \frac{x^4}{\sqrt[3]{y}}$

10. $2 \ln x + \frac{1}{2} \ln y - \ln z - 3 \ln w$

$\ln x^2 + \ln y^{1/2} - (\ln z + \ln w^3)$
$= \ln \frac{x^2\sqrt{y}}{zw^3}$

Expand each natural logarithm.

11. $\ln x^3\sqrt{y}$

$= 3\ln x + \ln y^{1/2}$

$= 3\ln x + \frac{1}{2}\ln y$

12. $\ln \frac{a^2b^3}{c\sqrt{d}}$

$= \ln a^2 + \ln b^3 - (\ln c + \ln d^{1/2})$

$= 2\ln a + 3\ln b - \ln c - \frac{1}{2}\ln d$

13. $\ln \frac{x^3}{e^2} = \ln x^3 - \ln e^2$

$= 3\ln x - 2\overset{1}{\cancel{\ln e}}$

$= 3\ln x - 2$

Applications of Natural Logarithms

Unit10 Day9

Warm-up:
Solve and round to the nearest hundredth

$$1600 = 4e^{.045t}$$

$$\frac{1600}{4} = \frac{4e^{.045t}}{4}$$

$$400 = e^{.045t}$$

$$\frac{\ln 400}{.045} = \frac{.045t \cancel{\ln e}}{.045}$$

$$t = 133.14$$

Money compounded continuously: $A = Pe^{rt}$

Where A = Ending amount

P = Principle (starting amount)

r = rate as a decimal

t = time

Exponential Growth or Decay: $N(t) = N_0e^{kt}$ Populations / continuous growth

Where $N(t)$ = Ending amount

N_0 = Starting amount

k = growth/decay constant (given as a decimal)

t = time

Examples:

1. A certain city has a population of $P(t) = 142{,}000e^{0.014t}$ where t is the time in years and t = 0 is the year . 2010

2010

a. What is the population in ? 142,000

b. What is the population in ?2020 $P(10) = 142{,}000e^{.014(10)} = 163{,}339$

c. In what year will the city have a population of 200,000?

$$\frac{200{,}000}{142{,}000} = \frac{142{,}000e^{.014t}}{142{,}000}$$

$$\frac{200}{142} = e^{.014t}$$

$$\frac{\ln\left(\frac{200}{142}\right)}{.014} = \frac{.014t\ \cancel{\ln e}}{.014}$$

t = 24.46 years after 2010

During 2034.

2. Sam invested a sum of money in a certificate of deposit that pays 8% interest compounded continuously. If he made the investment on January 1, 1997 and the account was worth \$10,000 on January 1, 2016, what was the original amount in the account?

$r = .08$

A

$t = 19$ yrs

P

$A = Pe^{rt}$

$$\frac{10{,}000}{e^{.08(19)}} = \frac{Pe^{.08(19)}}{e^{.08(19)}}$$

$P = \$2{,}187.12$

3. Mike deposited some money in a bank account that earns 5.6% interest compounded continuously. How long will it take to double the money in his account?

$r = .056$

t

$P = 1$
$A = 2$

$A = Pe^{rt}$

$2 = \cancel{1}e^{.056t}$

$$\frac{\ln 2}{.056} = \frac{.056t \cancel{\ln e}}{.056}$$

$t = 12.4$ yrs

4. DDT is an insecticide that has been used by farmers. It decays slowly and is sometimes absorbed by plants that animals and humans eat. DDT absorbed in the mud at the bottom of a lake is degraded into harmless products by bacterial action. ~~Experimental data shows that 10% of the initial amount is eliminated in 5 years.~~ ~~If~~ k = -0.0211

a. How much of the original amount of DDT is left after 10 years?

Start = 100% $N(t) = 100e^{-.0211(10)} = 80.774$ ~81%

b. The US Environmental Protection Agency banned almost all use of DDT in the US in 1972. If none has been used near the lake since then, in what year will the concentration of DDT fall below 25%?

$$\frac{25}{100} = \frac{100e^{-.0211t}}{100}$$

$$.25 = e^{-.0211t}$$

$$\frac{\ln(.25)}{-.0211} = \frac{-.0211t \, \cancel{\ln e}}{-.0211}$$

$t = 65.7$ years after 1972

1972+65 = During 2037