# Unit 10 Day 10

1 (a) $A(t) = 600e^{-.243t}$

(b) See graph on next slide

(c) $t = 7.4$ hours

4 $D = 6.3$ Volts

2 (4)

3 (a) $T = 38°C$

(b) 66 minutes

(In the 65 minute)

Name________________________________ Alg 2 CC

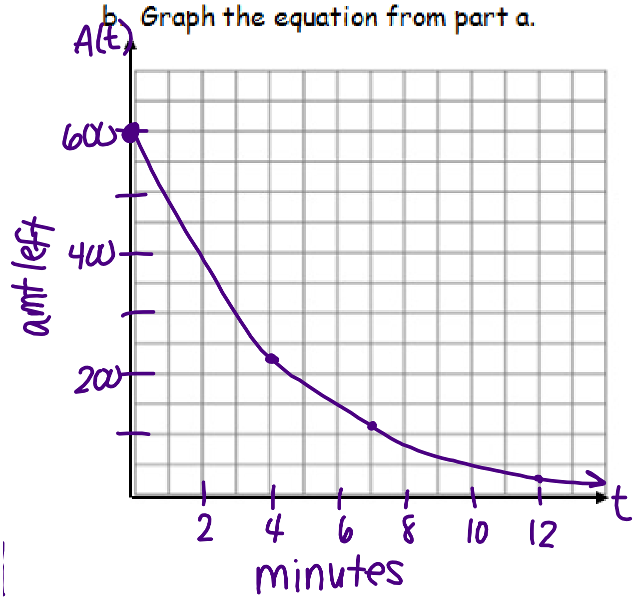
Date____________________ Unit10 Day10HW

1. Medications break down in the human body at different rates. The breakdown of a certain medication is represented by the function $A(t) = A_0(e)^{-rt}$, where $A(t)$ is the amount left in the body, $A_0$ is the initial dosage, $r$ is the decay rate, and $t$ is the time in hours. A patient is given 600 milligrams of a certain medication with a decay rate of 0.243.

a. Write the equation for $A(t)$ that represents the breakdown of the medication.

$A(t) = \underline{600e^{-.243t}}$

b. Graph the equation from part a.




| t | 0 | 4 | 7 | 12 |
|---|---|---|---|---|
| A(t) | 600 | 227 | 109.5 | 32.49 |

x-min 0
x-max 10
y-min 0
y-max 700

c. It is safe to take another dose of the medication when you have only 100 milligrams left in your system. Determine, to the *nearest tenth of an hour*, how long a person needs to wait to take another dose of the medication.

$$\frac{100}{600} = \frac{600e^{-.243t}}{600}$$

$$\frac{1}{6} = e^{-.243t}$$

$$\ln\frac{1}{6} = -.243t$$

$$t = \frac{\ln\frac{1}{6}}{-.243}$$

$t \sim 7.4$ hours

2. Franco invests $4,500 in an account that earns a 3.8% nominal interest rate compounded continuously. If he withdraws the profit from the investment after 5 years, how much has he earned on his investment?

(1) $858.92 (3) $922.50

(2) $912.59 (4) $941.62

$A = 4500e^{.038(5)}$

$A = \$5441.62$

$5441.62 - 4500 = \$941.62$

3. A cup of water at an initial temperature of $76^0C$ is placed in a room at a constant temperature of $20^0C$. As the water cools, its temperature is described by the equation $T = 20 + 56e^{-0.037t}$, where t is the time elapsed in minutes.

a. What is the temperature of the water one-half hour after the cup was placed in the room, to the nearest degree Celsius?

$t = 30$

$T = 20 + 56e^{-.037(30)}$

$T = 38°$

b. How many minutes will it take for the water to cool off to $25^0C$?

$25 = 20 + 56e^{-.037t}$

$-20 \quad -20$

$\frac{5}{56} = \frac{56e^{-.037t}}{56}$

$\frac{\ln\left(\frac{5}{56}\right)}{-.037} = \frac{-.037t \cancel{\ln e}}{-.037}$

$t = 66$ minutes

4. The power output $P_0$ of an amplifier is given by the formula $P_0 = P_i e^{D/10}$, where $P_i$ is the power input and D is the decibel voltage gain. Determine the decibel voltage gain, to the *nearest tenth*, for an amplifier with a power output of 60W and an input power of 32W.

$$\frac{60}{32} = \frac{32e^{D/10}}{32}$$

$$10 \cdot \ln\left(\frac{60}{32}\right) = \frac{D}{10} \overset{1}{\cancel{\ln e}} \cdot 10$$

$$D = 6.3$$

The decibel voltage gain is 6.3.

# Review for Unit 10 Test

# Answers to the Unit 10 review

1. $y = 4^x$
2. $\{64\}$
3. d
4. $y = \log_2(x-4)+2$
5. $2 + \frac{1}{3}\log x - 2\log y$
6. $\{4\}$
7. $\{\frac{3}{2}\}$
8. $\{1.98\}$
9. $\{2.681\}$
10. $\ln(a^2\sqrt[3]{b})$
11. $\ln 2 + \ln x - \ln y - 3\ln z$
12. $\log_3 9 = 2$
13. $\{\frac{20}{11}\}$
14. $\{.99\}$
15. $\{4.98\}$
16. $\{.7\}$
17. See work on slide
18. 48.78
19. $A = 4200e^{.03t}$
20. $B(t) = 10000(\frac{1}{2})^{t/3}$
21. 52.3 years
22. 33.8 years
23. $k = .0858$

Name_______________________________ Alg 2 CC Review for Unit 10 2019

Date________________

1. What is the inverse of $y = \log_4 x$?

$4^y = x$

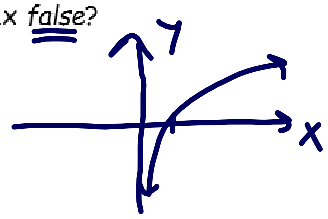
$y = 4^x$

2. If $\log_x 4 = 1/3$, what is the value of $x$?

$\log_x 4 = \frac{1}{3}$ $x^{\frac{1}{3} \cdot 3} = 4^3$ $\{64\}$

$x = 64$

3. Which statement about the graph of $c(x) = \log_4 x$ *false*?
   a. The domain is the set of positive reals.
   b. The graph contains the point (1,0).
   c. The graph has an asymptote at x=0.
   d. The graph has a y-intercept.



4. The graph of $y = \log_2 x$ is translated to the right 4 units and up 2 units. Write the equation of the translated graph.

$-4$ $+2$

$y = \log_2(x-4)+2$

5. Expand $\log \frac{100x^{\frac{1}{3}}}{y^2}$ $= \log 100 + \log x^{1/3} - \log y^2$

$= 2 + \frac{1}{3}\log x - 2\log y$

6. Evaluate log 10000.

Use calculator

$\{4\}$

7. Evaluate $\log_{25} 125$

Use calculator or $\log_{25} 125 = x$

$\{3/2\}$

$25^x = 125 \rightarrow 5^{2x} = 5^3$ $2x = 3$

$x = 3/2$

8. Evaluate and round to the nearest hundredth.

$\log_7 47$

$\{1.98\}$

9. Evaluate and round to the nearest thousandth.

$\log_{15} 1421$

$\{2.681\}$

10. Rewrite as a single natural logarithm.

2 ln a + 1/5 ln b

$\ln a^2 b^{1/5} = \ln a^2 \sqrt[5]{b}$

11. Expand the natural logarithm.

$\ln \frac{2x}{yz^3} = \ln 2 + \ln x - (\ln y + \ln z^3)$

$= \ln 2 + \ln x - \ln y - 3\ln z$

Step 1 Step 2

12. Rewrite as a single logarithm and evaluate.

$\frac{1}{3} \log_3 27 + \frac{1}{2} \log_3 9 = \log_3 27^{1/3} + \log_3 9^{1/2}$

$= \log_3 \sqrt[3]{27} \cdot \sqrt{9} = \log_3 3 \cdot 3$

$= \log_3 9 = 2$

13. Solve for x using common bases:

$243^{3x} = 81^{x+5}$

$3^{5(3x)} = 3^{4(x+5)}$

$15x = 4x + 20$
$-4x \quad -4x$
$\frac{11x}{11} = \frac{20}{11}$ $\quad x = \frac{20}{11}$ $\quad \{\frac{20}{11}\}$

14. Solve for x. Round to the nearest hundredth.

$8^{3x} = 475$

$\frac{\log_8 475}{3} = \frac{3x}{3}$
$x = .99$

or

$\frac{3x\log 8}{3\log 8} = \frac{\log 475}{3\log 8}$
$x = .99$ $\quad \{.99\}$

15. Solve for x. Round to the nearest hundredth.

$4^{x-2} = 62$

$\log_4 62 = x - 2$
$x = \log_4 62 + 2$
$x = 4.98$

or

$\frac{(x-2)\log 4}{\log 4} = \frac{\log 62}{\log 4}$
$x - 2 = \frac{\log 62}{\log 4} + 2$
$+2$
$x = 4.98$ $\quad \{4.98\}$

16. Solve for x using natural logarithms. Round your answer to the nearest tenth.

$5e^{2x} + 6 = 26$

$-6 \quad -6$

$\frac{5e^{2x}}{5} = \frac{20}{5}$

$e^{2x} = 4$

$\frac{\log_e 4}{2} = \frac{2x}{2}$ or $\frac{2x \cancel{\ln e}}{2} = \frac{\ln 4}{2}$

$x = .7$ $\quad x = \frac{\ln 4}{2} = .7$

$\{.7\}$

17. Graph the function $f(x) = \log_2(x-1)+2$

State the transformation(s) that occur.

Right 1, up 2

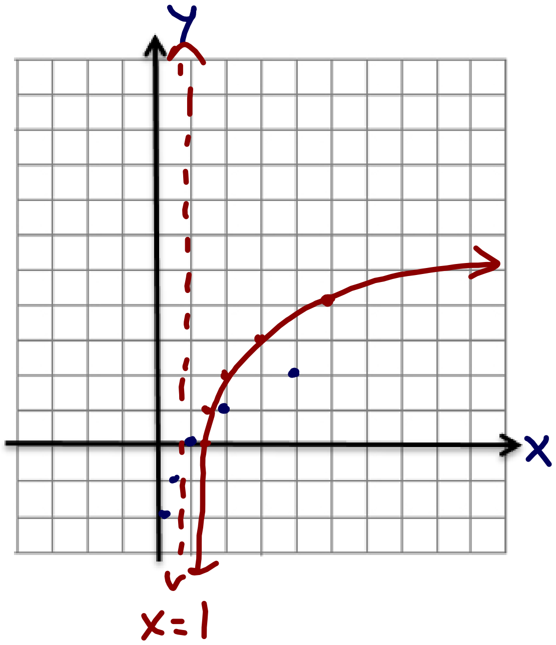
Domain $(1, \infty)$ or $\{x \mid x > 1\}$

Range $(-\infty, \infty)$ or $\{y \mid y \in \mathbb{R}\}$

Asymptote $x = 1$

State the end behavior as x approaches 1.

y approaches $-\infty$




$y = 2^x$

| x | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | 1/4 | 1/2 | 1 | 2 | 4 |

$y = \log_2 x$

| x | 1/4 | 1/2 | 1 | 2 | 4 |
|---|---|---|---|---|---|
| y | -2 | -1 | 0 | 1 | 2 |

move pts right 1 and up 2

18. The world population was 2560 million people in 1950 and 3040 million in 1960 and can be modeled by the function $p(t) = 2560e^{0.017185t}$, where t is time in years after 1950 and p(t) is the population in millions. Determine the average rate of change of p(t) in millions of people per year, from $4 \le t \le 8$. Round your answer to the *nearest hundredth.*

$P(8) = 2560e^{.017185(8)} = \underline{2937.2896}$

$P(4) = 2560e^{.017185(4)} = \underline{2742.1636}$

$\frac{\Delta y}{\Delta x} = \frac{2937.2896 - 2742.1636}{8-4} \approx \frac{195.126}{4} \approx \underline{48.78}$

19. You put \$4200 in a savings account paying 3% interest compounded continuously. Write an equation to model this situation.

$A = Pe^{rt}$

$A = 4200e^{.03t}$

20. A certain strain of bacteria has been reduced by half every 3 hours by a new medication being tested by the FDA. Write a function that gives the number of cells that contain the bacteria if there were 10,000 cells to start.

base exponent

$$B(t) = 10000\left(\tfrac{1}{2}\right)^{t/3}$$

21. You put \$1200 in a savings account paying 2.1% interest compounded continuously. How long will it take for your savings to triple? Round your answer to the nearest tenth of a year.

$A = Pe^{rt}$

$$A = 1200e^{.021t} \qquad 1200 \rightarrow 3600$$

$$\frac{3600}{1200} = \frac{1200e^{.021t}}{1200}$$

$$3 = e^{.021t}$$

$$\frac{\log_e 3}{.021} = \frac{.021t}{.021} \quad \text{or} \quad \frac{\ln 3}{.021} = \frac{.021t\,\overset{1}{\cancel{\ln e}}}{.021}$$

$$t = 52.3 \text{ years}$$

22. Your investment has been decreasing at a steady rate of 3.2% per year. If you originally invested \$3000, using the formula $A = a(1 \pm r)^t$, determine the number of years algebraically that it will take for your investment to reach \$1000. Round your answer to the nearest tenth of a year.

$A = 3000(1 - .032)^t$

$A = 3000(.968)^t$

$\frac{1000}{3000} = \frac{3000(.968)^t}{3000}$

$\frac{1}{3} = .968^t \rightarrow \log_{.968}(\frac{1}{3}) = t$ or $\frac{\log(\frac{1}{3})}{\log .968} = \frac{t \log .968}{\log .968}$

$t = 33.8$ years

23. In 2005, the deer population in Central New York was estimated to be 102,541. After a study done in 2015, it was estimated that the deer population grew to 241,730. Determine the rate of growth using the equation $N = N_0e^{kT}$. Round to the nearest ten-thousandths place.

time
2015
-2005
10 years

$241{,}730 \div 102{,}541 = e^{10t}$

$\frac{241{,}730}{102{,}541} = e^{10k}$

$\frac{\log_e\left(\frac{241730}{102541}\right)}{10} = \frac{10k}{10}$

$k = .0858$

or $\frac{\ln\left(\frac{241730}{102541}\right)}{10} = \frac{10k \cancel{\ln e}}{10}$

$k = .0858$

$\{.0858\}$