HW 11-1 $-5+7(n-1)$

1) $a_n = 7n - 2$; $a_1 = 5$, $a_n = a_{n-1} + 7$; 40
2) not arithmetic
3) $a_n = \frac{14}{3} - \frac{2}{3}n$, $a_1 = \frac{12}{3}$, $a_n = a_{n-1} - \frac{2}{3}$; $\frac{2}{3}$
4) f(n)= $5n - 2$, 38
5) f(n)= -3.0 - .2n, -4.6
6) 15, 18
7) .6, -.2
8) -13
9) -17
10) (2)
11) 43
12a) A(n) = 175 - 2.75n or $A(n) = 172.25 - 2.75(n-1)$
   b) 63 weeks
   c) Each movie rental cost $2.75, so I divided 175 by 2.75 to see how many rentals w possible. $63.\overline{63}$ told me 63 rentals are possible, but not 64.

Name Key

Alg 2 HW **11-1**

Determine if the sequence is arithmetic. If so, find the common difference, an explicit n^{th} term formula (a_n), a recursive formula and the next term.

1. 5, 12, 19, 26, 33, ...

$d = 7$

$a_n = 5 + 7(n-1)$

E: $a_n = 7n - 2$

R: $a_n = a_{n-1} + 7,\ a_1 = 5$

$a_6 = 7(6) - 2 = 42 - 2 = 40$

2. 26, 19, 13, 8, 4, ...

$-7\ -6\ -5\ -4$

not arithmetic

3. $\frac{12}{3}, \frac{10}{3}, \frac{8}{3}, \frac{6}{3}, \frac{4}{3}, \ldots$

$d = -2/3$

$a_n = \frac{12}{3} - \frac{2}{3}(n-1)$

$a_n = \frac{12}{3} - \frac{2}{3}n + \frac{2}{3}$

E $a_n = \frac{14}{3} - \frac{2}{3}n$

R $a_n = a_{n-1} - \frac{2}{3},\ a_1 = \frac{12}{3}$

$a_6 = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$

Write an explicit formula in function notation for each arithmetic sequence and use it to find the 8^{th} term.

4. 3, 8, 13, 18, ...

$d = 5$

$f(n) = 3 + 5(n-1)$

$f(n) = 3 + 5n - 5$

$f(n) = 5n - 2$

$f(8) = 5(8) - 2 = 40 - 2 = 38$

5. -3.2, -3.4, -3.6, -3.8, ...

$d = -.2$

$f(n) = -3.2 - .2(n-1)$

$f(n) = -3.2 - .2n + .2$

$f(n) = -3.0 - .2n$

$f(8) = -3.0 - .2(8) = -3.0 - 1.6 = -4.6$

Find the missing ~~terms~~ in each arithmetic sequence. * Didn't model in notes, see what they can do. Good discussion & practice for regents style.

*6. 12, 15, 18, 21

a_1, a_2, a_3, a_4

$d = \frac{a_4 - a_1}{4-1} = \frac{21-12}{3} = \frac{9}{3} = 3$

$12 + 3 = 15$

$15 + 3 = 18$

$18 + 3 = 21$ ✓

7. 1.4, .6, -.2, -1

$d = \frac{a_4 - a_1}{4-1} = \frac{-1-1.4}{3} = \frac{-2.4}{3} = -.8$

$1.4 - .8 = .6$

$.6 - .8 = -.2$

$-.2 - .8 = -1$ ✓

Use a formula to find the 9^{th} term of each arithmetic sequence.

8. $a_4 = 27$ and $a_5 = 19$

$d = a_5 - a_4 = 19 - 27 = -8$

$a_n = a_1 + d(n-1)$

$a_9 = a_5 - 8(9-5)$

$a_9 = 19 - 8(4) = 19 - 32$

$a_9 = -13$

9. $a_3 = -5$ and $a_6 = -11$

$d = \frac{a_6 - a_3}{6-3} = \frac{-11+5}{3} = \frac{-6}{3} = -2$

$a_n = a_1 + d(n-1)$

$a_9 = a_6 - 2(9-6)$

$a_9 = -11 - 2(3) = -11 - 6 = -17$

10. The third term in an arithmetic sequence is 10 and the fifth term is 26. If the first term is a_1, which is an equation for the nth term of this sequence?

$a_3 = 10$, $a_5 = 26$

1) $a_n = 8n + 10$
2) $a_n = 8n - 14$
3) $a_n = 16n + 10$
4) $a_n = 16n - 38$

$d = \frac{a_5 - a_3}{5-3} = \frac{26-10}{2} = \frac{16}{2} = 8$

$a_3 = 8 \cdot 3 - 14 = 10$

$a_5 = 8 \cdot 5 - 14 = 26$

$a_5 = a_3 + 2d$

$a_1 = a_3 - 2d$

$a_1 = 10 - 2(8)$

$a_1 = -6$

$a_n = -6 + 8(n-1)$

$= 8n - \underbrace{6 - 8}_{-14}$

#24 Aug AI CC Regents

11. If $f(1) = 3$ and $f(n) = -2f(n-1) + 1$, then find $f(5)$. Recursive

3, -5, -21

$f(2) = -2f(2-1) + 1 = -2f(1) + 1 = -2(3) + 1 = -6 + 1 = -5$

$f(3) = -2(-5) + 1 = 10 + 1 = 11$

$f(4) = -2(11) + 1 = -22 + 1 = -21$

$f(5) = -2(-21) + 1 = 42 + 1 = 43$

Not Arithmetic

#35 Jun 2014 AI CC Regents

12. Caitlin has a movie rental card worth \$175. After she rents the first movie, the card's value is \$172.25. After she rents the second movie, its value is \$169.50. After she rents the third movie, the card is worth \$166.75. Assuming the pattern continues,
a) write an equation to define $A(n)$, the amount of money on the rental card after n rentals. b) Caitlin rents a movie every Friday night. How many weeks in a row can she afford to rent a movie, using her rental card only? c) Explain how you arrived at your answer.

172.25, 169.50, 166.75

$d = 172.25 - 175 = -2.75$

$A(n) = 172.25 - 2.75(n-1)$

a) $A(n) = 172.25 - 2.75n + 2.75$

$\boxed{A(n) = 175 - 2.75n}$

b) Original worth = \$175

$175 \div 2.75 = 63.63$

63 weeks

c) Each movie rental cost 2.75 so I divided 175 by 2.75 to see how many rentals were possible. 63.63 told me 63 rentals are possible, but not 64.

Geometric Sequences Day 2

Warm-up:

In 1202, Italian mathematician Leonardo Fibonacci described how fast rabbits breed under ideal circumstances. Fibonacci looked at the number of pairs of rabbits each month and formed the famous pattern called the Fibonacci sequence. It is recursively defined as:

any term, the term before, the term before that

$f(n) = f(n-1) + f(n-2)$ and $f(1) = f(2) = 1$

a) Generate values for: f(3), f(4), f(5) and f(6) which are the next four terms of this sequence.

$f(3) = f(2) + f(1) = 1 + 1 = 2$

$f(4) = f(3) + f(2) = 2 + 1 = 3$

$f(5) = f(4) + f(3) = 3 + 2 = 5$

$f(6) = f(5) + f(4) = 5 + 3 = 8$

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

b) Describe the number pattern generated by the Fibonacci sequence.

Each term is equal to the sum of the two previous terms.

c) Explain why this sequence is not arithmetic.

There is no common difference.

Definition:

A Geometric Sequence is a sequence in which the ratio of successive terms is a constant, called the common ratio (r and r ≠ 1).

$r = \frac{a_2}{a_1}$

64, 32, 16, 8, 4... **r = 32/64 = 1/2 or 0.5**

In this geometric sequence, each term is 1/2 the previous term.

1. Determine whether each of the following sequences could be arithmetic, geometric or neither. If possible, find the common difference or common ratio.

G a. -1, 1/2, -1/4, 1/8... $r = -1/2$

A b. 1, 1/2, 0, -1/2... $d = -1/2$

N c. -8, -4, -2, -1, 0...

2. Determine if each sequence is geometric. If so, find r.

E a. $a_n = 5(3)^{n-1}$

$a_1 = 5(3)^{\cancel{1-1}\,0} = 5(1) = 5$

$a_2 = 5(3)^{2-1} = 15$

$a_3 = 5(3)^{3-1} = 5(9) = 45$

G $r=3$

b. $a_1 = 2$ and $a_n = -2a_{n-1}$

R

$a_1 = 2$

$a_2 = -2a_1 = -2(2) = -4$

$a_3 = -2(-4) = 8$

$a_4 = -2(8) = -16$

G $r=-2$

Geometric Sequence Recursive Definition:

any term the term before × r

Given f(1), then f(n) = f(n-1) • r, where r is called the common ratio.
Note r can be positive or negative and often is fractional.

3. Generate the next three terms of the geometric sequences given below.

a. $a_1 = 4$ and $r = 2$

$a_1 = 4$
$a_2 = 4(2) = 8$
$a_3 = 8(2) = 16$
$a_4 = 16(2) = 32$

b. $t_n = t_{n-1} \cdot \sqrt{2}$ with $t_1 = 3\sqrt{2}$

$t_1 = 3\sqrt{2}$
$t_2 = 3\sqrt{2} \cdot \sqrt{2} = 3(2) = 6$
$t_3 = 6\sqrt{2}$
$t_4 = 6\sqrt{2} \cdot \sqrt{2} = 6(2) = 12$

4. Like arithmetic sequences, we need to be able to determine any given term of a geometric sequence based on the first value, the common ratio and the term number. Consider $a_1 = 2$ and $a_n = a_{n-1} \cdot 3$

a_n

$r = 3$

a. Generate the value of a_4.

$a_2 = a_1 \cdot 3 = 2(3) = 6$

$a_3 = a_2 \cdot 3 = 6(3) = 18$

$a_4 = a_3 \cdot 3 = 18(3) = 54$

b. How many times did you need to multiply 2 by 3 in order to find a_4?

3

$(n-1)$ times

c. Determine the value of a_{10} without generating a_9.

$a_n = 2(3)^{n-1}$

d. Write a formula for the n^{th} term of a geometric sequence, a_n, based on the first term, a_1, r and n.

Geometric Sequence Explicit Definition:
$a_n = a_1 r^{n-1}$

A: $a_n = a_1 + d(n-1)$

5. Find the 9th term of the following geometric sequences.

$a_n = a_1 r^{n-1}$

a. 0.001, 0.01, 0.1, 1...

$r = \frac{a_2}{a_1} = \frac{.01}{.001} = 10$

$a_n = .001(10)^{n-1}$

$a_9 = .001(10)^{9-1} = 100,000$

b. $\frac{3}{4}, \frac{-3}{8}, \frac{3}{16}, \frac{-3}{32}...$

$r = \frac{-1}{2}$

$a_n = \frac{3}{4}\left(\frac{-1}{2}\right)^{n-1}$

$a_n = \frac{3}{4}\left(\frac{-1}{2}\right)^{9-1} = \frac{3}{1024}$

6. Find the 10th term of the geometric sequence.

a. $a_4 = -8$ and $a_6 = -200$

$\underset{a_4}{\_\_} \xrightarrow{\times r} \underset{a_5}{\_\_} \xrightarrow{\times r} \underset{a_6}{\_\_}$

$a_6 = a_4 r^2$

$-200 = -8r^2$

$\sqrt{25} = \sqrt{r^2}$

$r = \pm 5$

$a_{10} = a_6 r^4$

$a_{10} = -200(\cancel{\pm 5})^4$ (625) $= -125{,}000$

b. $a_2 = 768$ and $a_4 = 48$ (find the 10th term)

$a_4 = a_2 r^2$

$\frac{48}{768} = \frac{768 r^2}{768}$

$\sqrt{\frac{1}{16}} = \sqrt{r^2}$

$r = \pm\frac{1}{4}$

$a_{10} = a_4 r^6$

$a_{10} = 48\left(\pm\frac{1}{4}\right)^6 = \frac{3}{256}$