HW 11-2

1) 5
2) Arithmetic, $d = -2$
3) Neither
4) Geometric, $r = \frac{1}{4}$
5) 39,366
6) .000005
7) 64
8) $-\frac{4}{3} \text{ or } -1.\overline{3}$
9) 324
10) 8, -8, 8
11) 24, 36, 54
12) -10, 20, -40
13a) \$33,600; \$35,280
b) 1.05
c) $a_n = a_{n-1} \bullet 1.05$
d) $a_n = 32{,}000(1.05)^{n-1}$
e) \$49,642.50, I used the explicit rule so I could get the 10 year salary directly by using $n = 10$. I would have had to use the recursive formula several times to get that.

Name Key Alg 2 HW 11-2

1. If a sequence is defined recursively by $f(0) = 2$ and $f(n+1) = -2f(n) + 3$ for $n \geq 0$, then find $f(2)$.

$n = 0 \rightarrow f(0+1) = f(1) = -2f(0) + 3 = -2(2) + 3 = -1$

$n = 1 \rightarrow f(1+1) = f(2) = -2f(1) + 3 = -2(-1) + 3 = 5$

$\therefore f(2) = 5$

For 2 - 4, determine if each sequence is arithmetic, geometric, or neither. If possible, find the common difference or ratio.

2. $\underbrace{-10, -12}_{-2}, \underbrace{-14}_{-2}, \underbrace{-16}_{-2}, \ldots$

Arithmetic, $d = -2$

3. $\frac{1}{2}, 1, 2, 3, \ldots$

Neither

4. $-320, -80, -20, -5, \ldots$ ($\times \frac{1}{4}$ $\times \frac{1}{4}$ $\times \frac{1}{4}$)

Geometric, $r = \frac{1}{4}$

For 5 - 7, use an explicit formula to find the 10th term of each geometric sequence.

5. $2, 6, 18, 54, 162, \ldots$

$r = 3$

$a_{10} = 2(3)^9$

$a_{10} = 2(19683)$

$a_{10} = 39{,}366$

6. $5000, 500, 50, 5, 0.5, \ldots$

$r = \frac{500}{5000} = \frac{1}{10}$

$a_{10} = 5000\left(\frac{1}{10}\right)^9$

$a_{10} = .000005$

7. $-0.125, 0.25, -0.5, 1, -2, \ldots$

$r = \frac{.25}{-.125} = -2$

$a_{10} = -.125(-2)^9 = 64$

For 8 & 9, use a formula to find the 6th term of the geometric sequence with the given terms.

8. $a_4 = -12$, $a_5 = -4$

$r = \frac{a_5}{a_4} = \frac{-4}{-12} = \frac{1}{3}$

$a_6 = a_5\left(\frac{1}{3}\right)$

$a_6 = -4\left(\frac{1}{3}\right) = \frac{-4}{3}$

or $-1.\overline{3}$

9. $a_2 = 4$, $a_5 = 108$

Find r

$a_n = a_1 r^{n-1}$

$a_5 = a_2 r^{5-2}$

$\frac{108}{4} = \frac{4r^3}{4}$

$\sqrt[3]{r^3} = \sqrt[3]{27}$

$r = 3$

$a_6 = a_2(3)^{6-2}$

$a_6 = 4(3)^4$

$a_6 = 4(81)$

$a_6 = 324$

For 10 - 12, generate the next three terms of each geometric sequence.

10. $a_1 = -8$ with $r = -1$

$a_2 = -8(-1) = 8$
$a_3 = 8(-1) = -8$
$a_4 = -8(-1) = 8$

11. $a_n = a_{n-1} \cdot \frac{3}{2}$ and $a_1 = 16$

$a_2 = a_1 \cdot \frac{3}{2} = 16 \cdot \frac{3}{2} = 24$
$a_3 = 24 \cdot \frac{3}{2} = 36$
$a_4 = 36 \cdot \frac{3}{2} = 54$

12. $f(n) = f(n-1) \cdot -2$ and $f(1) = 5$

$f(2) = f(1) \cdot -2 = 5 \cdot -2 = -10$
$f(3) = -10 \cdot -2 = 20$
$f(4) = 20 \cdot -2 = -40$

13. A math teacher earns \$32,000 in his first year of teaching. Each successive year, he earned a 5% raise.

100% + 5% = 105%

a) Find his salary for year 2 and year 3.

year 2 = 32000 + .05(32000) = \$33600
year 3 = 33600 + .05(33600) = \$35280

b) Find the common ratio for this geometric sequence.

$r = \frac{33600}{32000} = 1.05$

c) Write a recursive rule, a_n, in terms of a_{n-1} and given a_1 = \$32,000.
✓ check your formula by getting the next 2 terms. Do they match your part a answers?

$a_n = a_{n-1} \cdot r \longrightarrow a_n = a_{n-1} \cdot 1.05$

d) Write an explicit rule, a_n, in terms of $n - 1$. ✓ Check your formula by getting the next 2 terms. Do they match part a?

$a_n = a_1 r^{n-1}$
$a_n = 32000(1.05)^{n-1}$

e) Determine his salary for year 10. Which rule did you use? Why?

$n = 10$
$a_{10} = 32000(1.05)^9$
$a_{10} = \$49,642.50$

I used my explicit rule so I could get the 10 year salary directly by using $n = 10$. I would have had to use the recursive formula several

Summation Notation **Day 3**

Summation Notation can be used to represent the sum of any sequence of numbers (not just arithmetic or geometric sequences) which can be defined by a rule (formula).

The sum of a sequence is called a series.

1, 2, 4, 8, 16... is a geometric sequence

1 + 2 + 4 + 8 + 16 is a geometric series

SUMMATION (SIGMA) NOTATION

$$\sum_{k=a}^{n} f(k) = f(a) + f(a+1) + f(a+2) + \ldots f(n)$$

where k is defined as the index variable, which starts at a value of a, ends at a value of n, and increases by 1 each time.

For example, the series 1 + 2 + 4 + 8 + 16 can also be written as follows:

$$\sum_{k=0}^{4} 2^k = 2^0 + 2^1 + 2^2 + 2^3 + 2^4$$

$$1 + 2 + 4 + 8 + 16 = 31$$

1. Expand each series and evaluate.

a. $\sum_{k=3}^{5} 2k$

$2(3)+2(4)+2(5)$

$6+8+10$

24

b. $\sum_{n=-1}^{3} n^2$

$(-1)^2+0^2+1^2+2^2+3^2$

$1+0+1+4+9$

15

c. $\sum_{i=0}^{2} \frac{1}{2^i}$

$\frac{1}{2^0}+\frac{1}{2^1}+\frac{1}{2^2}$

$1+\frac{1}{2}+\frac{1}{4}=\frac{7}{4}$ or 1.75

d. $\sum_{k=1}^{5} (-1)^k$

$(-1)^1+(-1)^2+(-1)^3+(-1)^4+(-1)^5$

$\cancel{-1+1}\;\cancel{-1+1}-1$

-1

e. $3\sum_{m=0}^{2} (2m-1)$

$3\left[2(0)-1+2(1)-1+2(2)-1\right]$

$3(\cancel{-1+1}+3)=9$

f. $\sum_{k=1}^{3} -5(2)^{k-1}$

$-5(\cancel{2})^{\cancel{1-1}}_{1} + -5(\cancel{2})^{\cancel{2-1}}_{2} + -5(\cancel{2})^{\cancel{3-1}}_{4}$

$-5 + -10 + -20$

-35

2. Consider the sequence defined recursively by $a_n = a_{n-1} + 2a_{n-2}$ and $a_1 = 0$ and $a_2 = 1$. Find the value of:

$$\sum_{n=3}^{5} a_n = a_3 + a_4 + a_5$$
$$= 1 + 3 + 5 = 9$$

$a_3 = a_2 + 2a_1 = 1 + 2(0) = 1$

$a_4 = a_3 + 2a_2 = 1 + 2(1) = 3$

$a_5 = a_4 + 2a_3 = 3 + 2(1) = 5$

3. Express each sum using summation or sigma notation and use n as your index variable.

$n = 1 \ldots 8$

a) -4 + -1 + 2 + 5 + ... + 17

$+3 \quad +3 \quad +3 \quad 8 \quad 11 \quad 14$

$a_n = a_1 + d(n-1)$

$a_n = -4 + 3(n-1)$

$$\boxed{\sum_{n=1}^{8} (-4 + 3(n-1))}$$

b) $\frac{1}{25} + \frac{1}{5} + 1 + 5 + \ldots + 625$

$\times 5 \quad \times 5 \quad 25 \quad 125$

$n = 1 \quad 2 \quad 3 \quad 4 \ldots 7$

$a_n = a_1 r^{n-1}$

$a_n = \frac{1}{25}(5)^{n-1}$

$$\boxed{\sum_{n=1}^{7} \left(\frac{1}{25}(5)^{n-1}\right)}$$

c) 9 + 16 + 25 + ... + 100

$1 \quad 2 \quad 3$

$$\sum_{n=3}^{10} n^2 \quad \text{or} \quad \sum_{n=1}^{8} (n+2)^2$$

#4

Not choice 4 → way too many terms

1) → $3^1 + \cancel{3}^{2}$ 9

2) → $\cancel{3(2)^0}^{\,3} + \cancel{3(2)^1}^{\,6} + \ldots \cancel{3(2)^4}^{\,48}$

3) → $\cancel{6}^{\,1/6}\ {}^{0-1}$

4. Which of the following represents the sum of 3 + 6 + 12 + 24 + 48?

(1) $\sum_{k=1}^{5} 3^k$

(3) $\sum_{k=0}^{4} 6^{k-1}$

(2) $\sum_{k=0}^{4} 3(2)^k$

(4) $\sum_{k=3}^{48} k$