Alg 2 Sequence & Series Test Review

Sequence & Series Test Review Key

1a. $d = \frac{1}{8}$, Arithmetic; b. Neither; c. r=-3, Geometric

2a. 4, 2, 1, $\frac{1}{2}$, Recursive; b. 1, -3, -7, -11, Explicit

3. -14

4. 12, -12

5. 8000

6. 125

7. f(n) = -19 + 6n

8. $a_n = 7\left(\frac{1}{4}\right)^{n-1}$ or $a_n = \frac{7}{4^{n-1}}$

9. -7

10. (c)

11. $\sum_{n=1}^{8} \frac{1}{27}(3)^{n-1}$ or $\sum_{n=-3}^{4} 3^n$ or $\sum_{n=1}^{8} 3^{n-4}$

12. 236.35

13a. $S_n = \frac{2500(1-1.05^n)}{1-1.05}$ or $S_n = \frac{2500-2500(1.05^n)}{1-1.05}$

13b. \$70,331

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Arithmetic Sequence: $a_n = a_1 + d(n - 1)$

Geometric Sequence: $a_n = a_1(r)^{n-1}$

Geometric Series

$$S_n = \frac{a_1(1-r^n)}{1-r} = \frac{a_1 - a_1 r^n}{1-r}, \text{ where } r \neq 1$$

1. Determine whether each of the following sequences could be arithmetic, geometric or neither. If possible, find the common difference or common ratio.

   a. 7/8, 1, 9/8, 5/4...

   $d = 1 - \frac{7}{8} = \frac{1}{8}$

   Arithmetic

   b. 9, 3, 1, 0...

   $r = \frac{1}{3}$?

   No. $1 \times \frac{1}{3} = \frac{1}{3} \neq 0$

   Neither

   c. .2, -.6, 1.8, -5.4...

   $r = \frac{-.6}{.2} = -3$

   Geometric

2. Find the first four terms of the sequence and tell if the formula is explicit (E) or recursive (R).

   a. $a_1 = 4$ and $a_n = \frac{1}{2}(a_{n-1})$

   $a_1 = 4$

   $a_2 = \frac{1}{2}(4) = 2$

   $a_3 = \frac{1}{2}(2) = 1$

   $a_4 = \frac{1}{2}(1) = \frac{1}{2}$

   R

   b. $a_n = 5 - 4n$

   $a_1 = 5 - 4(1) = 1$

   $a_2 = 5 - 4(2) = 5 - 8 = -3$

   $a_3 = 5 - 4(3) = 5 - 12 = -7$

   $a_4 = 5 - 4(4) = 5 - 16 = -11$

3. A sequence is defined by $a_n = 2a_{n-1} + a_{n-2}$ with $a_1 = 3$ and $a_2 = -4$. Determine the value of a_4.

$a_3 = 2(a_2) + a_1 = 2(-4) + 3 = -8 + 3 = -5$

$a_4 = 2(a_3) + a_2 = 2(-5) + -4 = -10 - 4 = -14$

4. Find the missing term in this geometric sequence: 36, ___, 4.

a_1, a_2, a_3

$a_n = a_1(r)^{n-1}$

$a_3 = a_1(r)^{3-1}$

$\frac{4}{36} = \frac{36(r)^2}{36}$

$r^2 = \frac{4}{36}$

$\sqrt{r^2} = \pm\sqrt{\frac{1}{9}}$

$r = \pm\frac{1}{3}$

$a_2 = 36(\pm\frac{1}{3})$

$a_2 = \pm 12$

12 or -12

5. Stacks of ice blocks are formed to represent a pattern. The first 4 stacks have the following number of blocks: 1, 8, 27, 64. How many cubes are in the 20th stack? Justify your answer.

$1^3, 2^3, 3^3, 4^3, \ldots 20^3$

$= 8000$

6. A sequence is defined using the rule $a_n = 5a_{n-1} + 1$ with $a_1 = 6$. Find the value of $a_3 - a_2$.

$a_2 = 5(a_1)+1 = 5(6)+1 = 31$

$a_3 = 5(a_2)+1 = 5(31)+1 = 156$

$a_3 - a_2 = 156 - 31 = 125$

7. $f(n) = f(n-1) + 6$ and $f(1) = -13$ represents a recursive formula. Write an explicit formula for the same sequence.

$f(2) = f(1) + 6 = -13+6 = -7$) +6

$f(3) = f(2) + 6 = -7+6 = -1$) +6

$f(4) = f(3) + 6 = -1+6 = 5$

Arithmetic, $d = 6$

$f(n) = f(1) + d(n-1)$

$f(n) = -13 + 6(n-1)$

$f(n) = -13 + 6n - 6$

$\boxed{f(n) = -19 + 6n}$

8. A geometric sequence is given below. Find an explicit formula for the nth term of this sequence.

$7, \frac{7}{4}, \frac{7}{16}, \frac{7}{64}, \frac{7}{256}, \ldots$

$r = \frac{\frac{7}{4}}{\frac{7}{1}} = \frac{7}{4} \cdot \frac{1}{7} = \frac{1}{4}$

$r = \frac{1}{4}$, $a_1 = 7$

$a_n = a_1 r^{n-1}$

$a_n = 7\left(\frac{1}{4}\right)^{n-1}$

or

$a_n = \frac{7}{4^{n-1}}$

9. If $f(x) = \sum_{n=2}^{4}(n^2 + xn)$, then $f(-4) =$ $\sum_{n=2}^{4}(n^2 + {-4}n) =$

$= \underbrace{(2^2 - 4(2))}_{n=2} + \underbrace{(3^2 - 4(3))}_{n=3} + \underbrace{(4^2 - 4(4))}_{n=4}$

$= (4 - 8) + (9 - 12) + (16 - 16)$

$= -4 + -3 = -7$

10. Which of the following represents the series $\frac{2}{9} + \frac{3}{16} + \frac{4}{25} + \frac{5}{36} + \frac{6}{49}$?

~~(a)~~ $\sum_{n=3}^{7}\frac{n-1}{n+6}$ $4+6=10 \neq 16$

~~(b)~~ $\sum_{n=3}^{7}\frac{n-1}{n^2+1}$ $3^2+1=10 \neq 9$

(c) $\sum_{n=2}^{6}\frac{n}{(n+1)^2}$

~~(d)~~ $\sum_{n=2}^{6}\frac{n}{n^2+1}$ $2^2+1=5 \neq 9$

$\to \underset{n=2}{\frac{2}{9}} + \underset{n=3}{\frac{3}{16}} + \underset{n=4}{\frac{4}{25}} + \underset{n=5}{\frac{5}{36}} + \underset{n=6}{\frac{6}{49}}$ yes.

11. Express the following sum using summation (sigma) notation.

$$\frac{1}{27}+\frac{1}{9}+\frac{1}{3}+1+3+9+27+81$$

$r = \frac{3}{1} = 3$

$a_n = a_1 r^{n-1}$

$a_n = \frac{1}{27}(3)^{n-1}$

$$\boxed{\sum_{n=1}^{8} \frac{1}{27}(3)^{n-1}}$$

or $3^{-3}+3^{-2}+3^{-1}+3^{0}+3^{1}+3^{2}+3^{3}+3^{4}$

$$\boxed{\sum_{n=-3}^{4} 3^{n} \quad \text{or} \quad \sum_{n=1}^{8} 3^{n-4}}$$

12. Determine the sum of a geometric series with 12 terms whose first term is 130 and whose common ratio is 0.45. Round to the nearest hundredth. Use the formula.

$n = 12,\ a_1 = 130,\ r = .45$

$S_n = \frac{a_1(1-r^n)}{(1-r)}$ $\quad S_{12} = \frac{130(1-.45^{12})}{(1-.45)} = 236.35$

13. Nicky is having a baby and the baby's grandparents plan to set up an education fund account to make deposits annually starting on the baby's first birthday. The first deposit will be $2500 with each deposit being increased by 5% each year.

a. Write a geometric series formula, S_n, for the total deposited amount over n years.

$$a_1 = 2500,$$
$$r = 1 + .05 = 1.05$$

$$S_n = \frac{2500(1-1.05^r)}{1-1.05}$$

b. Find the account's total deposited amount to the nearest dollar at the end of 18 years.

$$S_{18} = \frac{2500(1-1.05^{18})}{1-1.05} = 70330.9616.. \approx \$70,331$$