Name KEY Algebra 2 Homework 16-1

1. The following segment lengths were selected in four different random samples of size 10.

Lengths Sample A	Lengths Sample B	Lengths Sample C	Lengths Sample D
1	1	1	2
2	3	5	2
1	1	1	7
5	2	3	2
3	1	4	5
1	5	2	2
2	3	2	3
2	4	4	5
3	3	3	5
1	3	4	4

a. Find the mean segments length of each sample and the standard deviation of each sample.

A: $\bar{x} = 2.1$ $S_x = 1.29$ B: $\bar{x} = 2.6$ $S_x = 1.35$ C: $\bar{x} = 2.9$ $S_x = 1.37$ D: $\bar{x} = 3.7$ $S_x = 1.77$

b. Find the mean and standard deviation of the four sample means ~~and standard deviations~~.

Put x in L1 Stat-Calc-1

Mean of sample means = 2.825

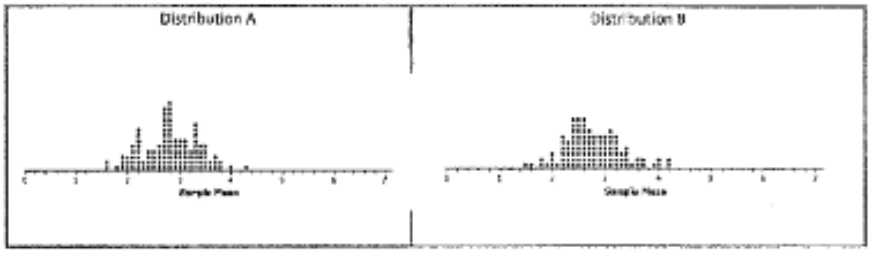
SD of sample means = .67

c. Interpret your answer to part (b) in terms of the variability in the sampling process.

A typical distance of a sample mean from the mean of the 4 samples (2.825) is within .67.

Sample would typically be between 2.155 and 3.495.

2. Two simulated sampling distributions of the mean segment lengths from random samples of size 10 are displayed below.



a. Compare the distributions with respect to shape, center, and spread.

Both distributions ~ symmetric w/ a center a bit below 3, about 2.8.

The max mean segment length in both N about 4.2 units & min around 1.5 or 1.6. Most of sample means in both distr. are betw. 2 and 4.

b. Distribution A has a mean of 2.82 and Distribution B has a mean of 2.77. How do these means compare to the population mean of 2.78?

The mean segment lengths of the 2 simulated distr. of sample means are very close to the actual mean segment length.

c. Both Distribution A and Distribution B have a standard deviation 0.54. Make a statement about the distribution of sample means that makes use of this standard deviation.

A typical distance of a sample mean from the center of the sampling distr. is 0.54.

Most of our data should be within .54 of the mean (2.78).

Margin of Error When Estimating a Population Proportion

Mystery Bag Activity:

A student will draw a chip from the bag and record whether the chip was red or not red. Return the chip to the bag, shake the bag, and another student will draw and record the color of the second chip. Continue this process until we have a sample of 30 chips.

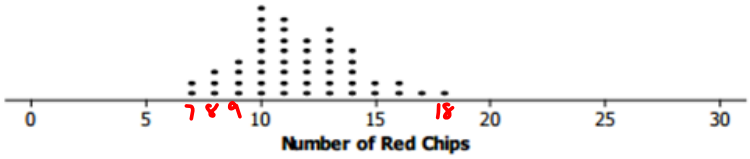
Trial	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Red?	y	N	N	N	N	N	y	y	N	y	N	N	N	y	y	y	y	N	N	N	N	N	N	N	N	N	Y	N	N	N

9 Red 21 Black

1. Write down your estimate for the proportion of red chips in the mystery bag based on the random sample of 30 chips drawn in class.

$P(\text{Red}) = \frac{9}{30} = .3$

2. Tanya and Ron had a paper bag that contained red and black chips. The bag was marked 40% red chips. They drew random samples of 30 chips, with replacement, from the bag. (They were careful to shake the bag after they replaced a chip.) They had nine red chips in their sample. They drew another random sample of 30 chips from the bag, and this time they had 12 red chips. They repeated this sampling process 50 times and made a plot of the number of red chips in each sample. A plot of their sampling distribution is shown below.



a. What was the most common number of red chips in the 50 samples? Does this seem reasonable? Why or why not? 10 red chips most common, seems reasonable.

Expect 30(0.4) = 12 red chips, 10 is close to that

b. What number of red chips, if any, never occurred in any of the samples?

Sample with <7 red chips or >18 red chips never occurred

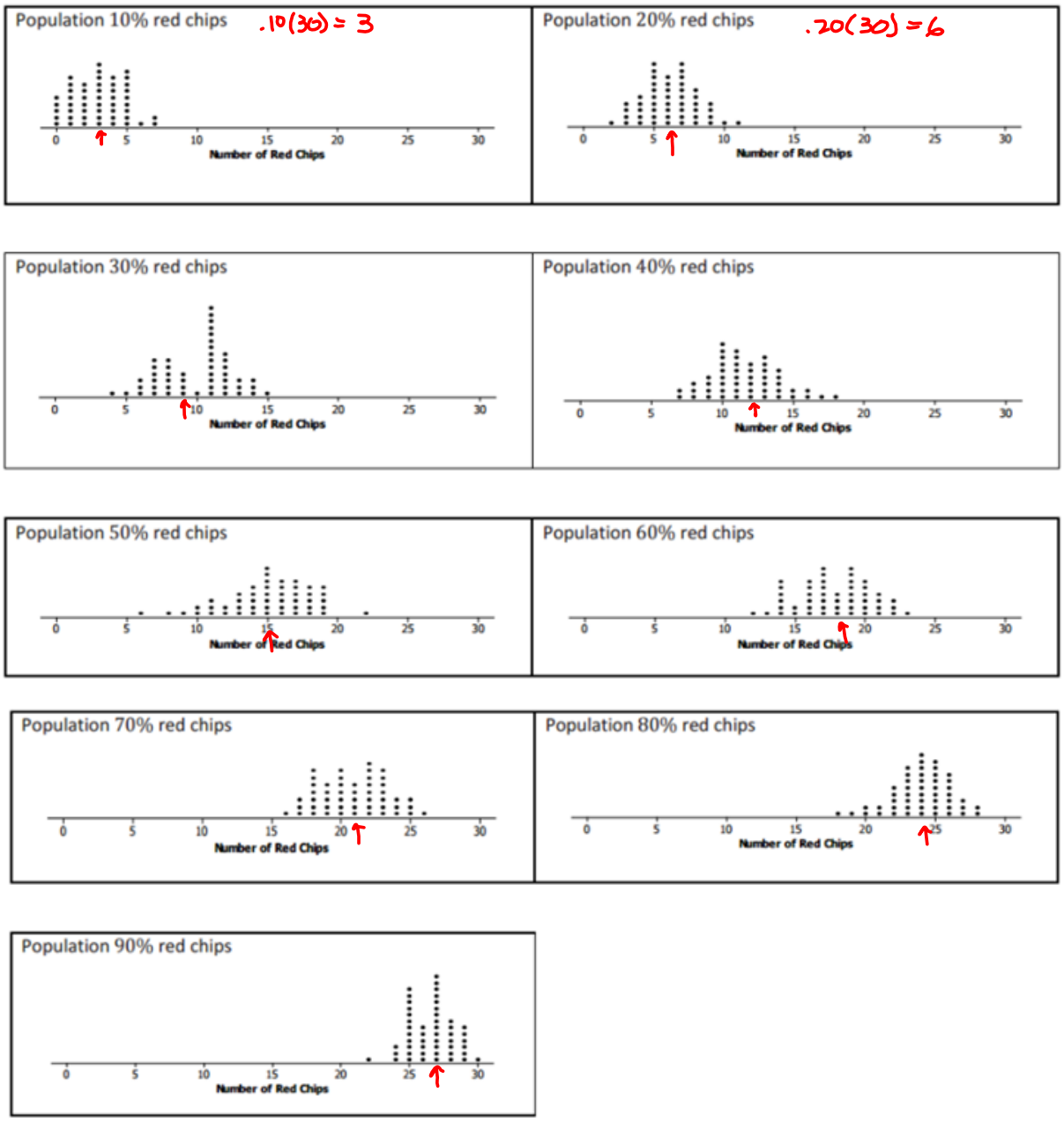
c. Give an interval that contains the "likely" (in this case possible) number of red chips in samples of size 30 based on the simulated sampling distribution.

7-18 red chips are likely (or possible)

d. Do you think the number of red chips in the (our) mystery bag could have come from a sample drawn from a bag that had 40% red chips? Why or why not?

Yes b/c 9 chips occurs in our sampling dist.

Nine different bags of chips were distributed to small teams of students in the class. Each bag had a different proportion of red chips. Each team simulated drawing 50 different random samples of size 30 from their bag and recorded the number of red chips for each sample (LOOK, REPLACE, SHAKE). The graphs of their simulated sampling distributions are shown below.



3. Think about the number of red chips in the random sample of size 30 that was drawn from our mystery bag at the start of the lesson.
   a. Based on the simulated sampling distributions, do you think that the mystery bag might have had 10% red chips? Explain your reasoning.

No - 9 Never occurs in the 10% sampling dist.

b. Based on the simulated sampling distributions, which of the percentages 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80% and 90% might reasonably be the percentage of red chips in the mystery bag? (In other words, look at the # of red chips pulled from our Mystery Bag in #1, which dot plots on the previous page INCLUDE our # of red chips-this gives us a possible % of red chips.)

20% to 50%

c. Let p represent the proportion of red chips in the mystery bag. (For example, p = 0.40 if there are 40% red chips in the bag.) Based on your answer to part (b), write an inequality that describes plausible values for p. Interpret the inequality in terms of the mystery bag population.

.20 ≤ p ≤ .50

Based on our sampling distribution, the true proportion of red chips in mystery bag could have been anywhere from .2 to .5. It wouldn't be surprising for a random sample of size 30 drawn from any of these populations to include 9 red chips.

95% Confidence Interval: A range of values defined such that there is a specified probability that a value of a parameter lies within the interval. (The probability that over many different confidence intervals, 95% of the confidence intervals will capture the true population mean/proportion). *See next slide for visual of CI

Confidence Interval = estimate ± Margin of Error ME = 2 S.D.

So... Confidence Interval = estimate ± 2 Standard Deviations

The margin of error is a range of values below and above a sample statistic in a confidence interval. The confidence interval is a way to show what the uncertainty is with a certain statistic. Proportions in the confidence interval are expected to occur when taking a sample from the mystery bag.

a. Write the confidence interval from Exercise 3 using this notation.

.2 to .5 ⟹ .35 ± .15

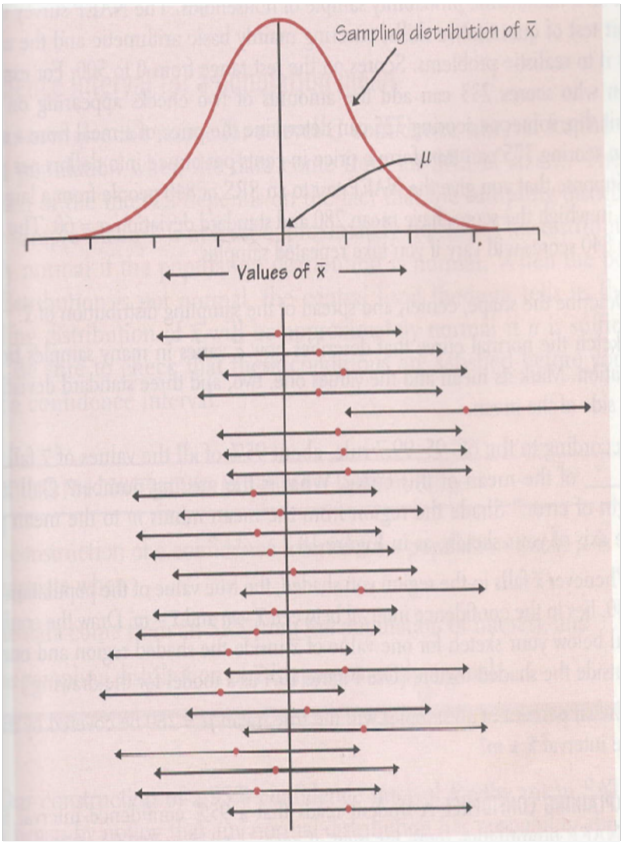
$\frac{\text{max}+\text{min}}{2} = \frac{.2+.5}{2} = .35$

$\frac{\text{max}-\text{min}}{2} = \frac{.5-.2}{2} = .15$

4. Do you think the "margin of error" would be different in exercise 4 if you had sampled 50 chips instead of 30? Try to convince a partner that your conjecture is correct.

↑SS causes ↓spread (↓ME, ↓SD)

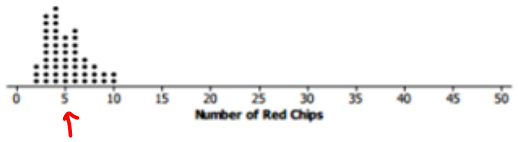
∴ ME should be lower



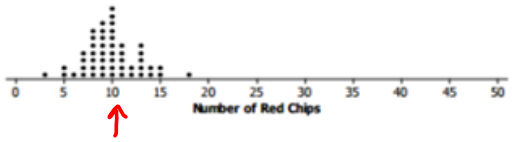
Twenty-five samples from the same population gave these 95% confidence intervals. In the long run, 95% of all samples give an interval that contains the population parameter μ.

5. Below are simulated sampling distributions of the number of red chips for samples of size 50 from populations with various percentages of red chips.

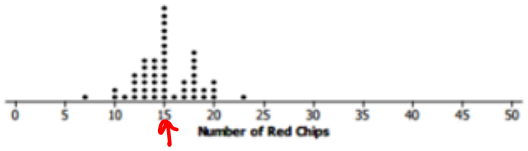
Population with **10**% red chips .10(50) = 5



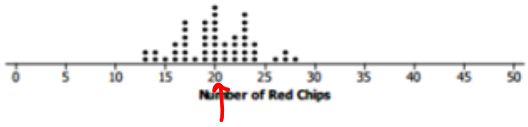
Population with **20**% red chips .20(50) = 10



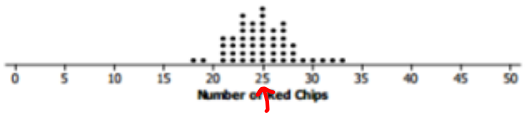
Population with **30**% red chips



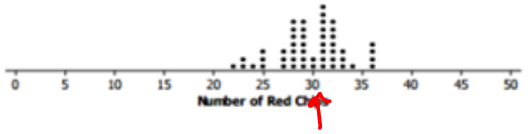
Population with **40**% red chips



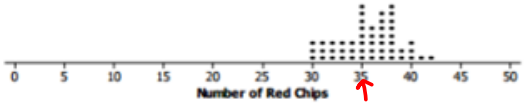
Population with **50**% red chips



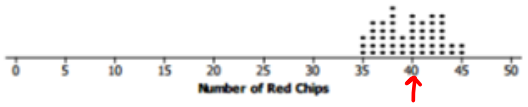
Population with **60**% red chips



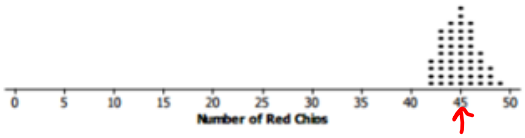
Population with **70**% red chips



Population with **80**% red chips



Population with **90**% red chips



a. Suppose you drew 30 red chips in a random sample of 50 from the mystery bag. What are plausible values for the proportion of red chips in the mystery bag? Explain your reasoning.

50% to 70% $.5 \leq p \leq .7$

30 Red chips occured in each of these sampling distributions.

b. Write an expression that contains the margin of error based on your answer to part (a).

Est. $= \frac{.5+.7}{2} = .6$ $\quad .6 \pm .1$

ME $= \frac{.7-.5}{2} = .1$ $\quad$ ↑ ME

6. Remember your conjecture from Exercise 5, and compare the margin of error you found for a sample of size 30 (from Exercise 3) to the margin of error you found for a sample of size 50.

a. Was your reasoning in Exercise 5 correct? Why or why not?

yes ME decreased from .15 to .10

b. Explain why the change in the margin of error makes sense.

Makes sense; as sample size increases, variability from sample to sample decreases. (Sample Prop. tend to be closer to actual population proportion)