

Name Key

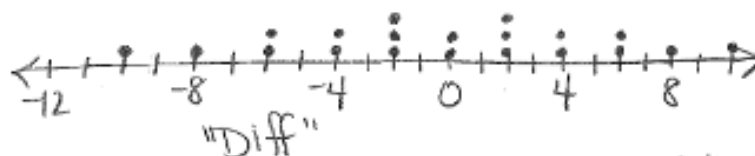
HW 14-8

Six ping-pong balls are labeled as follows: 0, 3, 6, 9, 12, 18. Three ping-pong balls will be randomly assigned to Group A; the rest will be assigned to Group B. "Diff" = $\bar{x}_A - \bar{x}_B$.

1. Develop the 20 possible random assignments to two groups, and calculate the "Diff" value for each. (Note: Avoid redundant cases; selecting "0, 3, and 6" for Group A is NOT a distinct random assignment from selecting "6, 0, and 3" so do not record both!)

| Group A Selection | Group B Selection | "A" Mean | "B" Mean | "Diff" |
|-------------------|-------------------|----------|----------|--------|
| 0 3 6 | 9 12 18 | 3 | 13 | -10 |
| 0 3 9 | 6 12 18 | 4 | 12 | -8 |
| 0 3 12 | 6 9 18 | 5 | 11 | -6 |
| 0 3 18 | 6 9 12 | 7 | 9 | -2 |
| 0 6 9 | 3 12 18 | 5 | 11 | -6 |
| 0 6 12 | 3 9 18 | 6 | 10 | -4 |
| 0 6 18 | 3 9 12 | 8 | 8 | 0 |
| 0 9 12 | 3 6 18 | 7 | 9 | -2 |
| 0 9 18 | 3 6 12 | 9 | 7 | 2 |
| 0 12 18 | 3 6 9 | 10 | 6 | 4 |
| 3 6 9 | 0 12 18 | 6 | 10 | -4 |
| 3 6 12 | 0 9 18 | 7 | 9 | -2 |
| 3 6 18 | 0 9 12 | 9 | 7 | 2 |
| 3 9 12 | 0 6 18 | 8 | 8 | 0 |
| 3 9 18 | 0 6 12 | 10 | 6 | 4 |
| 3 12 18 | 0 6 9 | 11 | 5 | 6 |
| 6 9 12 | 0 3 18 | 9 | 5 | 4 |
| 6 9 18 | 0 3 12 | 11 | 5 | 6 |
| 6 12 18 | 0 3 9 | 12 | 4 | 8 |
| 9 12 18 | 0 3 6 | 13 | 3 | 10 |

2. Create a dot plot that shows the 20 "Diff" values obtained from the 20 possible randomizations. By visual inspection, what is the mean and median value of the distribution?



The mean and median appear to be "0" due to the center symmetry.

3. Based on your dot plot, what is the probability of obtaining a "Diff" value of "8 or higher"?

$$\frac{2}{20} = .10 = 10\%$$

4. Would a "Diff" value of "8 or higher" be considered a difference that is likely to happen or one that is unlikely to happen? Explain.

Unlikely to happen since a diff. this large would happen only 10% of the time.

5. Based on your dot plot, what is the probability of obtaining a "Diff" value of "-2 or smaller"?

$$\frac{9}{20} = .45 = 45\%$$

6. Would a "Diff" value of "-2 or smaller" be considered a difference that is likely to happen or one that is unlikely to happen? Explain.

Likely to happen since a diff. this small would happen 45% of the time.

More Ruling Out Chance

Previously, you considered the random assignment of 10 tomatoes into two distinct groups of 5 tomatoes each called Group A and Group B. With each random assignment, you calculated "Diff" = $\bar{x}_A - \bar{x}_B$, the difference between the mean weight of the 5 tomatoes in Group A and the mean weight of the 5 tomatoes in Group B.

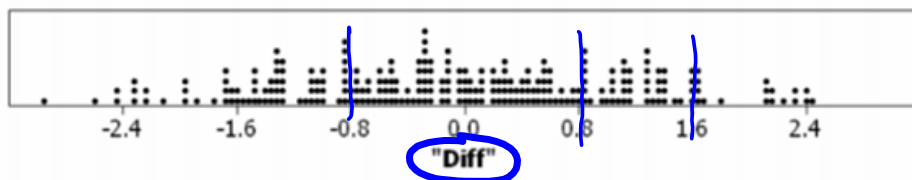
- a. Summarize in writing what you learned in the last lesson. Share your thoughts with a neighbor.

Single observ. divided into 2 groups, means of 2 groups differ by chance. In some cases means of 2 groups was large, others small ("0")

- b. Recall that 5 of these tomatoes are from plants that received a nutrient treatment in the hope of growing bigger tomatoes. But what if the treatment was not effective? What difference would you expect to find between the group means?

"Diff" = 0

In the previous lesson, 3 instances of the tomato randomization were considered. Imagine that the random assignment was conducted an additional 247 times, and 250 "Diff" values were computed from these 250 random assignments. The results are shown graphically below in a dot plot where each dot represents the "Diff" value that results from a random assignment:



This dot plot will serve as your randomization distribution for the "Diff" statistic in this tomato randomization example. The dots are placed at increments of 0.04 ounces.

- Given the distribution picture above, what is the approximate value of the median and mean of the distribution? Specifically, do you think this distribution is centered near a value that implies "No Difference" between Group A and Group B?

Mean & Median appear near "0," based on symmetry and center of distr. "Diff" value would be 0 (NO DIFFERENCE)

- Given the distribution pictured on the previous page and based on the simulation results, determine the approximate probability of obtaining a "Diff" value in the cases described in (a), (b), and (c).

a. of 1.64 ounces or more $\frac{17}{250} = .068 = 6.8\%$

- b. of -0.80 ounces or less (Hint: there should be 69)

$$\frac{69}{250} = 27.6\%$$

- c. within 0.80 ounces of 0 ounces (Hint: there should be 121)

$$\frac{121}{250} = 48.4\%$$

- d. How do you think these probabilities could be useful to people that are designing experiments?

Probability could be used to help determine if diff. occurred by chance or not.

In the context of a randomization distribution that is based upon the assumption that there is no real difference between the groups, consider a "Diff" value of X to be "statistically significant" if there is a low probability of obtaining a result that is as extreme as or more extreme than X .

3. Using that definition and your work above, would you consider any of the "Diff" values below to be statistically significant? Explain.

a. 1.64 ounces

Probably "stat. sign." Only 6.8% probability of occurring is not very frequent occurrence.

b. -0.80 ounces

Not "stat. sign." 27.6% probability of occurring is fairly common.

c. Values within 0.80 ounces of 0 ounces

Not "stat. sign." Not far from "0." Values close to "0" aren't "stat. sign." (Probability = 48.4% very likely)

4. In the previous lessons, you obtained "Diff" values of 0.28 ounces, 2.44 ounces, and 0 ounces for 3 different tomato randomizations. Would you consider any of those values to be "statistically significant" for this distribution? Explain.

Values 0 and 0.28 oz. wouldn't be "stat. sign." because not far from 0 in the distribution. 2.44 oz. would be "stat. sign." b/c it's far from 0 (only 4% chance of obtaining value that extreme or more in this distrib.) $\frac{1}{250}$

5. Recalling that "Diff" is the mean weight of the 5 Group A tomatoes minus the mean weight of the 5 Group B tomatoes, how would you explain the meaning of a "Diff" value of 1.64 ounces in this case?

The 5 tomatoes in Group A have a mean weight that's 1.64 oz. higher than Group B's mean weight.

Keep in mind that for reasons mentioned earlier, the randomization distribution above is demonstrating what is likely to happen by chance alone if the treatment was not effective. As stated in the previous lesson, you can use this randomization distribution to assess whether or not the actual difference in means *obtained from your experiment* (the difference between the mean weight of the 5 actual control group tomatoes and the mean weight of the 5 actual treatment group tomatoes) is consistent with usual chance behavior. The logic is as follows:

- If the observed difference is "extreme" and not typical of chance behavior, it may be considered "statistically significant" and possibly not the result of chance behavior.
 - If the difference is not the result of chance behavior, then maybe the difference did not just happen by chance alone.
 - If the difference did not just happen by chance alone, maybe the difference you observed is caused by the treatment in question, which, in this case, is the nutrient.
- In the context of our example, a statistically significant "Diff" value provides evidence that the nutrient treatment did in fact yield heavier tomatoes on average.

6. For reasons that will be explained in the next lesson, for your tomato example, "Diff" values that are positive and statistically significant will be considered as good evidence that your nutrient treatment did in fact yield heavier tomatoes on average. Again, using the randomization distribution shown earlier in the lesson, which (if any) of the following "Diff" values would you consider to be statistically significant and lead you to think that the nutrient treatment did, in fact, yield heavier tomatoes on average? Explain for each case.

- a. Diff = 0.4 ^{or higher} ~~near 0~~ $\frac{91}{250}$ } ^{rel. high} Not Stat Sign
 b. Diff = 0.8 ^{or higher} $\frac{64}{250}$ } [%] happen by chance
 c. Diff = 1.2 ^{or high} $\frac{38}{250}$ } ^{15%}
 d. Diff = 1.6 ^{or higher} $\frac{21}{250} \sim 8.4\%$ by chance } prob. starting to become stat. sign.
 e. Diff = 2.0 ^{or higher} $\frac{11}{250} \sim 4.4\%$ } } stat. sign.
 f. Diff = 2.4 ^{or higher} $\frac{3}{250} 1.2\%$ }

7. In the first random assignment in the previous lesson, you obtained a "Diff" value of 0.28 ounces. Earlier in this lesson, you were asked to consider if it might be a "statistically significant" value. Given the distribution shown in this lesson, if you had obtained a "Diff" value of 0.28 ounces *in your experiment* and the 5 Group A tomatoes had been the "treatment" tomatoes that received the nutrient, would you say that the "Diff" value was extreme enough to support a conclusion that the nutrient treatment yielded heavier tomatoes on average? Or, do you think such a "Diff" value may just occur by chance when the treatment is ineffective? Explain.

Not extreme enough to conclude that the nutrient trmt yielded heavier tomatoes on avg. Such a "Diff" may occur by chance.

0.28 is closer to 0 than other "not stat. sign." values.

8. In the second random assignment in the previous lesson, you obtained a "Diff" value of 2.44 ounces. Earlier in this lesson, you were asked to consider if this might be a "statistically significant" value. Given the distribution shown in this lesson, if you had obtained a "Diff" value of 2.44 ounces *in your experiment* and the 5 Group A tomatoes had been the "treatment" tomatoes that received the nutrient, would you say that the "Diff" value was extreme enough to support a conclusion that the nutrient treatment yielded heavier tomatoes on average? Or, do you think such a "Diff" value may just occur by chance when the treatment is ineffective? Explain.

Extreme enough to support a conclusion that nutrient trmt. yielded heavier tomatoes on avg. This didn't occur by chance alone. 2.44 is further from 0. (only 1/50 by chance)

Summary:

If the results appear typical of chance behavior and near the center of the distribution (that is, not relatively far from a "Diff" of 0), then there is little evidence that the treatment was effective. However, if it appears that the experiment's results are not typical of chance behavior, then, maybe, the difference you are observing didn't just happen by chance alone. It may indicate a statistically significant difference between the treatment group and the control group, and the source of that difference might be (in this case) the nutrient treatment.