Regents Review #5 - Polynomial Identities, Radicals, Complex Numbers

A **Polynomial Identity** is an equation that is true for all values of the replacement variable or variables. To prove an identity is true, we apply algebraic order of operations to one side of the equation to justify it is equivalent to the other side. We never move terms from one side to the other like when we solve an equation.

Algebraically prove the following where $x \neq -2$

a) Change only the right.

$$\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$$

$$\frac{x^3+8}{x^3+8} + \frac{1}{x^3+8}$$

$$\frac{x^3+9}{x^3+8}$$

$$\frac{x^3+9}{x^3+8} = \frac{x^3+9}{x^3+8} \checkmark$$

b) Do it again by changing only the left side.

$$\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$$

$$\frac{x^3+8}{x^3+8} + \frac{1}{x^3+8}$$

$$1 + \frac{1}{x^3+8} = 1 + \frac{1}{x^3+8} \checkmark$$

Solving Radical Equations: Review section 5 from your Review Packet.

Example: Solve algebraically for all values of x.

$$3 + \sqrt{7x-3} = x$$

$$(\sqrt{7x-3})^2 = (x-3)^2$$

$$(x-3)(x-3)$$

$$7x - 3 = x^2 - 6x + 9$$

$$0 = x^2 - 13x + 12$$

$$0 = (x-12)(x-1)$$

$x - 12 = 0$ | $x - 1 = 0$

$x = 12$ | ~~$x = 1$~~ reject

$\{12\}$

checks!

$$3 + \sqrt{7(12)-3} = 12$$

$$3 + \sqrt{81} = 12$$

$$12 = 12 \checkmark$$

$$3 + \sqrt{7(1)-3} = 1$$

$$3 + \sqrt{4} = 1$$

$$5 \neq 1$$

Radicals & Rational Exponents: Review section 12 from your Review Packet.

Use the properties of ~~rational~~ fractional exponents to determine the value of y for the equation:

$\boxed{\dfrac{\sqrt[3]{x^8}}{\left(x^4\right)^{\frac{1}{3}}}} = x^y, x > 1$

$$\frac{x^{8/3}}{x^{4/3}} = x^{4/3} = x^y$$

$$\frac{4}{3} = y$$

Operations With Complex #s: Review the top half of section 1 from your Review Packet.

Remember the Rules of i:

1. Change all expressions of the form $\sqrt{-b}$ to $i\sqrt{b}$ first.
2. Treat i as a variable for addition and subtraction.
3. Substitute -1 for i^2.

Example: Simplify $3i^2 - 6i^{12}$, where i is the imaginary unit.

$$3(-1) - 6(1)$$
$$-3 - 6 = -9$$

$$i^2 = -1$$
$$i^{12} = 1$$

Example: Simplify $xi(2i - 7i)^2$, where i is the imaginary unit.

$$xi(-5i)^2$$
$$xi(25\cancel{i^2}^{\,-1}) \quad \text{or} \quad 25x\cancel{i^3}^{\,-i}$$
$$xi(-25) \qquad -25xi$$
$$-25xi$$