HW 3 - 4: Answers

1. {(-1, 3), (-3, -1)}
2. {(1, 5), (5, -3)}
3. {(-7, 4), (4, 26)}
4. {(4, 3), (-4, -3)}
5. {(1, 3)}
6. x = 2, y = -2, z = 3

In 1 & 2: Solve graphically:

1 $y = 2x + 5$

2 $x^2 + y^2 - 4x + 2y - 20 = 0$

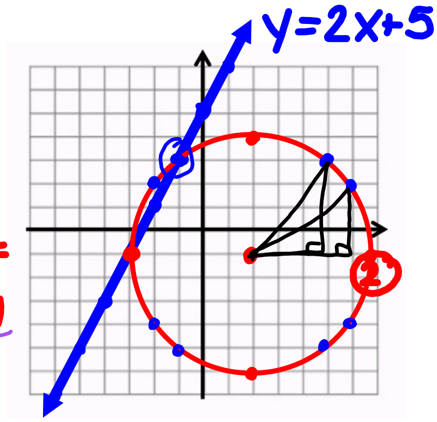
$m = 2$, $b = 5$

$x^2 - 4x + \underline{4} + y^2 + 2y + \underline{1} =$
$20 + \underline{4} + \underline{1}$

$(x-2)^2 + (y+1)^2 = 25$

center: $(2, -1)$

radius: 5



$\{(-1,3), (-3,-1)\}$

2. $y = -x^2 + 4x + 2$

$2x + y = 7$

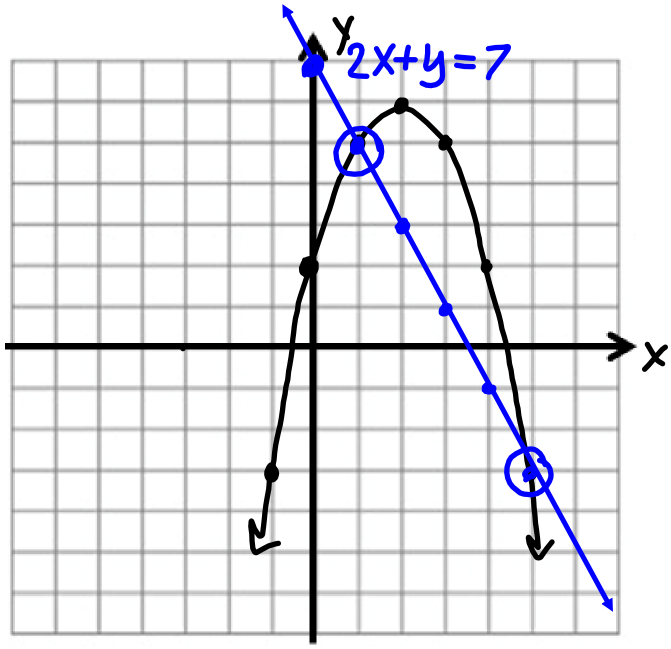
$y = -2x + 7$

$m = -2$

$b = 7$

x	-1	0	1	2	3	4	5
y	-3	2	5	6	5	2	-3

$\{(1,5), (5,-3)\}$



In 3 – 6, solve algebraically.

3. $y = x^2 + 5x - 10$
$y = 2x + 18$

$x^2+5x-10=2x+18$

$x^2+3x-28=0$

$(x+7)(x-4)=0$

$x=-7$	$x=4$
$y=2(-7)+18$	$y=2(4)+18$
$y=4$	$y=26$

$\{(-7,4),(4,26)\}$

4. $x^2 + y^2 = 25$
$y = \frac{3}{4}x$

$x^2 + \left(\frac{3}{4}x\right)^2 = 25$

$\frac{16}{16}x^2 + \frac{9}{16}x^2 = 25$

$\left(\frac{16}{25}\right)\frac{25}{16}x^2 = \cancel{25}\left(\frac{16}{\cancel{25}}\right)$

$x^2 = 16$

$x = \pm 4$

$x = 4$
$y = \frac{3}{4}(4)$
$y = 3$

$x = -4$
$y = \frac{3}{4}(-4)$
$y = -3$

$\{(4, 3), (-4, -3)\}$

5. $y = x^2 + 4x - 2$
$y = 6x - 3$

$x^2 + 4x - 2 = 6x - 3$

$x^2 - 2x + 1 = 0$

$(x - 1)^2 = 0$

$x - 1 = 0$ $\{(1, 3)\}$

$x = 1$

$y = 6(1) - 3$

$y = 3$

6. 1 $3x - y + z = 11 \longrightarrow \cdot 2 \Rightarrow 6x - 2y + 2z = 22$
2 $x + 4y - 2z = -12$
3 $2x + 2y - z = -3$

$$\begin{array}{r} 3x - y + z = 11 \\ 2x + 2y - z = -3 \\ \hline 5x + y = 8 \end{array} \qquad \begin{array}{r} 6x - 2y + 2z = 22 \\ x + 4y - 2z = -12 \\ \hline 7x + 2y = 10 \end{array}$$

$$\boxed{\begin{array}{l} x = 2 \\ y = -2 \\ z = 3 \end{array}}$$

$(2, -2, 3)$

$$\begin{array}{r} -2(5x + y = 8) \\ 7x + 2y = 10 \\ \hline -10x - 2y = -16 \\ 7x + 2y = 10 \\ \hline -3x = -6 \end{array}$$

$x = 2$

$7(2) + 2y = 10$

$2y = -4$

$y = -2$

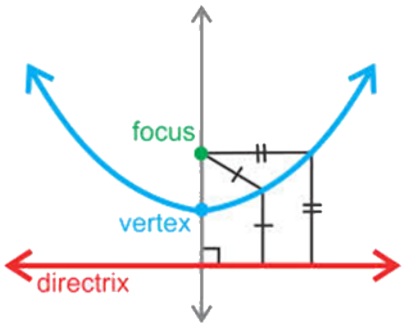
$3(2) + 2 + z = 11$

$z = 3$

U3D6

Parabolas

Parabola → The set of all points equidistant from a fixed point (focus) and a fixed line (directrix)



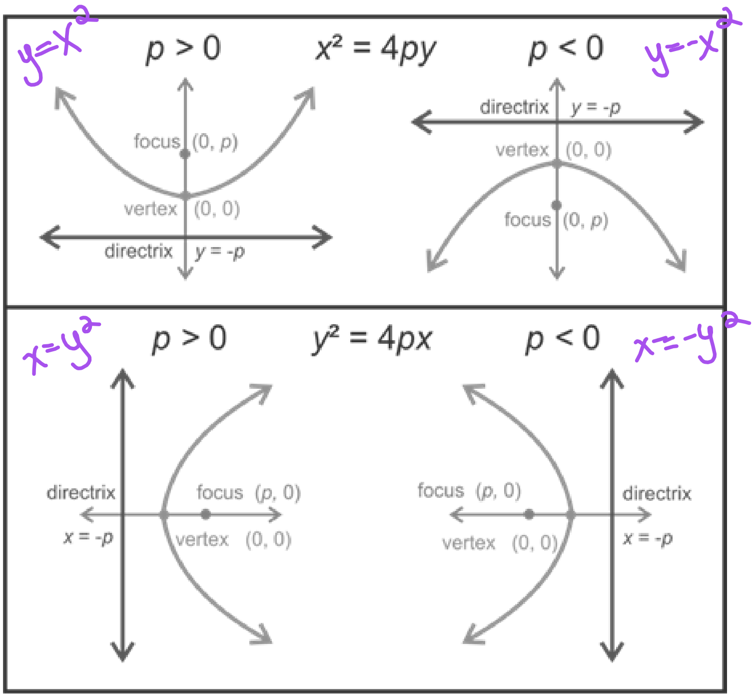
The **focus** is always **inside** the curve of the parabola.
The graph will always **bend away** from the **directrix**.
The **vertex** will always be **on the parabola** - right in the middle.
The **axis of symmetry** connects the focus and the vertex.

This is true no matter which way the parabola opens.

Equation (Standard Form):

$(x - h)^2 = 4p(y - k)$

$(y - k)^2 = 4p(x - h)$



For each of the images given, determine a possible equation for the parabola. Label the focus (F), vertex (V), directrix (y = or x =) and axis of symmetry (y = or x =).

$y=x^2$

$y^2=x$

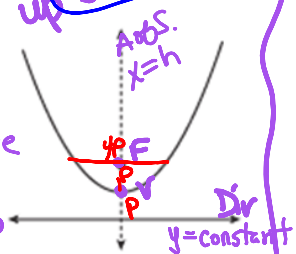
$(x-h)^2=4p(y-k)$

up

$p>0$

focus is p above vertex

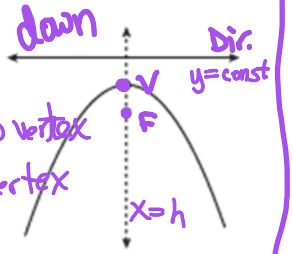
Dir. is p below vertex



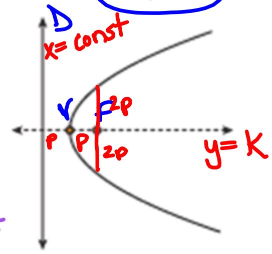
down

$p<0$

- focus is p below vertex
- Dir. is p above vertex

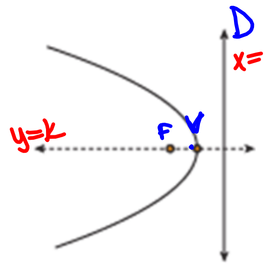


$(y-k)^2=4p(x-h)$



$p>0$

- focus is p to the rt. of vertex
- Dir is p to the left of vertex



$p<0$

- focus is p to the left of vertex
- Dir. is p to the rt. of vertex

Graph each of the parabolas given without the use of a calculator. Find and label all parts.

$(y-k)^2 = 4p(x-h)$

1. $(y - 2)^2 = 8(x + 1)$ left (Rt) $p>0$

$|4p| =$ 8 (4+4) LR

$\frac{4p}{4}=\frac{8}{4}$

$p =$ 2 Rt.

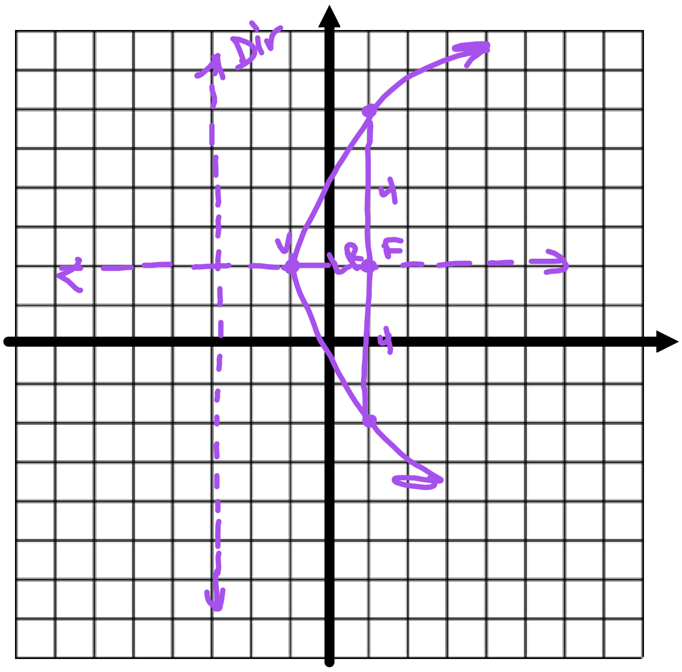
(h,k)

Vertex: (-1,2)

Focus: (1,2)

Directrix: $x=-3$

AOS: $y=2$



$(x-h)^2 = 4p(y-k)$ $y=x^2$ up down $p<0$

2. $(x - 1)^2 = -4(y + 3)$

$|4p|$ = -4 (2+2)

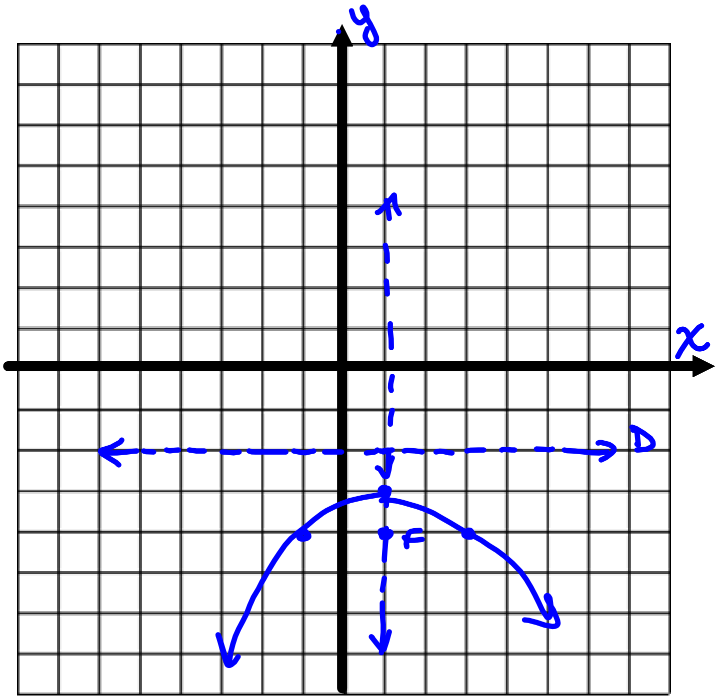
p = -1 (down 1)

Vertex: (1, -3)

Focus: (1, -4)

Directrix: $y = -2$

AOS: $x = 1$



3. $(y + 2)^2 = -2(x - 3)$

$|4p|$ = -2 (1+1)

x, y^2 left Rt $p<0$

$\frac{4p}{4}=\frac{-2}{4}$

p = $-\frac{1}{2}$ (left)

Vertex: (3, -2)

Focus: (2.5, -2)

Directrix: x = 3.5

AOS: y = -2