The number i

Algebra 2 Unit 4 Day 1

On your own, solve each equation for **x**.

1. $x - 1 = 0$ $x = 1$

2. $x + 1 = 0$ $x = -1$

3. $x^2 - 1 = 0$ $\sqrt{x^2} = \sqrt{1}$ $x = \pm 1$
$(x-1)(x+1) = 0$

4. $x^2 + 1 = 0$ $\sqrt{x^2} = \sqrt{-1}$ $\Longrightarrow$ $x = \pm i$
DNF $x = \pm\sqrt{-1}$

5. $x^2 + 2 = 0$ $\sqrt{x^2} = \sqrt{-2}$
$x = \pm\sqrt{-2}$ $\Longrightarrow$ $x = \pm i\sqrt{2}$

Which ones above do not have a real number solution? Why?

#4, 5 b/c you cannot take the square root of a negative number to get a real solution

In fact, solving the equation $x^2 + 1 = 0$, we got $x = \pm\sqrt{-1}$.

This leads to $i =$ $\sqrt{-1}$.

Problem: There is no real number that is the square root of a negative real number.

Solution: The number i.

We let $\sqrt{-1} =$ i, then $i^2 =$ -1. $(\sqrt{-1})^2 = -1$

If $r > 0$, $\sqrt{-r} = \sqrt{-1}\sqrt{r} = i\sqrt{r}$

Definition: A pure imaginary number is a number that can be written in the form bi where $b \neq 0$.

Rules of i:

1. Change all expressions of the form $\sqrt{-b}$ to $i\sqrt{b}$ first "Take the i out first"
2. Treat i as a variable for addition and subtraction.
3. Substitute -1 for i^2

Simplify:

1. $\sqrt{-9} = \sqrt{-1}\sqrt{9} = i\sqrt{9} = 3i$
2. $-\sqrt{-100} = -i\sqrt{100} = -10i$
3. $\sqrt{-20} = i\sqrt{20} = i\sqrt{4}\sqrt{5} = 2i\sqrt{5}$
4. $2\sqrt{-27} = 2i\sqrt{27} = 2i\sqrt{9}\sqrt{3} = 6i\sqrt{3}$

1
4
9
16
25
36
49
⋮

Note: In the real number system $\sqrt{a} \bullet \sqrt{b} = \sqrt{a \bullet b}$. However, this is not the case when working with imaginary numbers.

Example: Simplify the following using: a)rules for real numbers, and then b)rules for i.

a) $\sqrt{-4} \bullet \sqrt{-25}$

$\sqrt{100}$

10

b) $\sqrt{-4} \bullet \sqrt{-25}$

$i\sqrt{4} \cdot i\sqrt{25}$

$2i \cdot 5i$

$10i^2$

$10(-1)$

-10

① take i out 1st

② $i^2 = -1$

What do you notice?

Not =. Diff Rules for imaginary #'s.

Simplify:

5. $\sqrt{-9} \bullet \sqrt{-16} =$ $i\sqrt{9} \cdot i\sqrt{16} = 12i^2 = -12$

6. $\sqrt{5} \bullet \sqrt{-10} =$ $\sqrt{5} \cdot i\sqrt{10} = i\sqrt{50} = i\sqrt{25}\sqrt{2} = 5i\sqrt{2}$

7. $-\sqrt{-6} \bullet \sqrt{15} =$ $-i\sqrt{6} \cdot \sqrt{15} = -i\sqrt{90} = -i\sqrt{9}\sqrt{10} = -3i\sqrt{10}$

8. $(\sqrt{-7})^2 =$ $(i\sqrt{7})^2 = i^2(7) = -7$

Put everything away.

What are some things you learned about the number i today?