4-1 HW Answer Key

1. $\sqrt{-1}$
2. -1
3. $i\sqrt{r}$
4. 7i
5. -9i
6. $2i\sqrt{6}$
7. $6i\sqrt{5}$
8. -12
9. $2i\sqrt{15}$
10. $-3i\sqrt{5}$
11. -11
12. 50i
13. 2i
14. $-4i\sqrt{2}$
15. 12i
16. $x = \pm i\sqrt{5}$
17. $x = \pm 6i$
18. There is no real number that you can multiply by itself and get a negative number. For example, 2 • 2 = 4; -2 • -2 = 4.

Name Key Alg 2 Homework 4-1

1. $i = \sqrt{-1}$ 2. $i^2 = -1$ 3. $\sqrt{-r} = i\sqrt{r}$

Simplify:

4. $\sqrt{-49} = 7i$

5. $-\sqrt{-81} = -9i$

6. $\sqrt{-24} = i\sqrt{4}\sqrt{6} = 2i\sqrt{6}$

7. $2\sqrt{-45} = 2i\sqrt{9}\sqrt{5} = 6i\sqrt{5}$

8. $\sqrt{-4} \bullet \sqrt{-36} = 2i(6i) = 12i^2 = 12(-1) = -12$

9. $\sqrt{6} \bullet \sqrt{-10} = \sqrt{6} \cdot i\sqrt{10} = i\sqrt{60} = i\sqrt{4}\sqrt{15} = 2i\sqrt{15}$

10. $-\sqrt{-3} \bullet \sqrt{15} = -i\sqrt{3}\sqrt{15} = -i\sqrt{45} = -i\sqrt{9}\sqrt{5} = -3i\sqrt{5}$

11. $(\sqrt{-11})^2 = -11$

12. $5\sqrt{-100} = 5i\sqrt{100} = 5i(10) = 50i$

13. $\frac{1}{2}\sqrt{-16} = \frac{1}{2}i\sqrt{16} = \frac{1}{2}(4)i = 2i$

14. $-\sqrt{-32} = -i\sqrt{16}\sqrt{2} = -4i\sqrt{2}$

15. $\sqrt{-144} = 12i$

Solve for x and put in answer in i form.

16. $x^2 + 5 = 0$

$x^2 = -5$

$x = \pm i\sqrt{5}$

17. $x^2 + 36 = 0$

$x^2 = -36$

$x = \pm 6i$

18. Explain why there is no real number that is the square root of a negative number. For example, think about the $\sqrt{-4}$.

There's no real # that you can multiply by itself and get a negative #.

EX. $2(2) = 4$

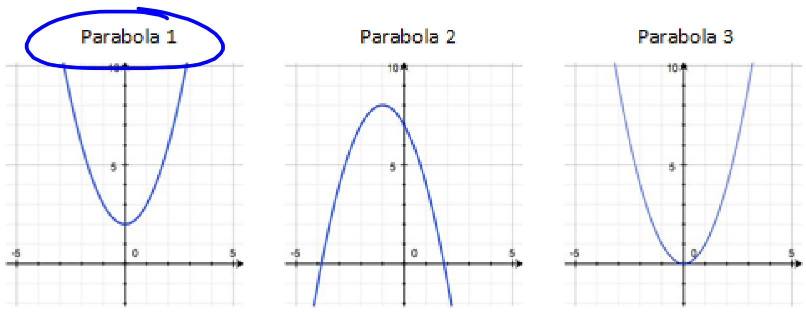
$-2(-2) = 4$

Complex numbers Algebra 2 Unit 4 Day 2

Yesterday we learned about a new number i. Today we are going to take it a step further and learn about complex numbers.

Which of these three parabolas are represented by a quadratic equation y= ax^2+bx+c that has no real solution to $ax^2 + bx + c = 0$? Explain.

Real solutions would be graphed as x-intercepts.
Parabola 1 has NO x-intercepts



2 Real solns.

1 Real soln.

Using the quadratic formula, $X = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, try solving $x^2 + 2x + 5 = 0$.

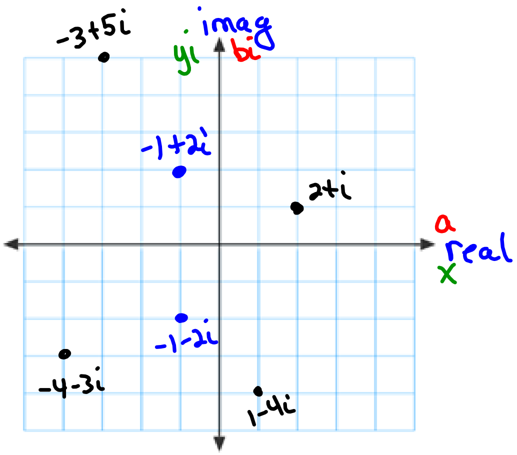
$a=1 \quad b=2 \quad c=5$

b^2-4ac

$4-4(1)(5) = -16$

$$X = \frac{-2 \pm \sqrt{-16}}{2(1)} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$$

These numbers are called Complex Numbers, which we can locate in the complex plane.



$-1 \pm 2i$

$-1+2i$

$-1-2i$

$2+i$

$-4-3i$

$-3+5i$

$1-4i$

In fact, all complex numbers ~~can~~ should be written in the form a + bi, where a and b are real numbers. Just as we can represent real numbers on the number line, we can represent complex numbers in the complex plane. Each complex number a+bi can be located in the complex plane in the same way we locate the point (a, b) in the Cartesian plane. From the origin, translate a units horizontally along the real axis and b units vertically along the imaginary axis.

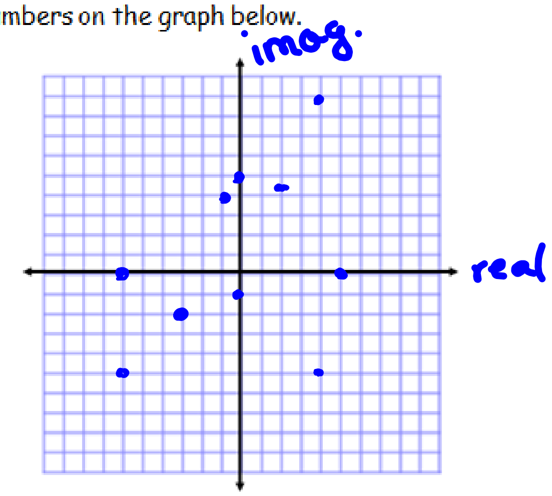
Are real numbers also complex numbers? Explain.

Yes. a Real number would have $b = 0$

Ex: $6 = 6 + 0i$

Plot and label the following complex numbers on the graph below.

1. -3 - 2i
2. -i
3. 5 + 0i
4. -1 + 4i
5. 2 + (9/2)i
6. 4 - 5i
7. 9i + 4   4+9i
8. -5i - 6   -6-5i
9. 5i
10. -6



Since complex numbers are built from real numbers, we should be able to add, subtract, multiply and divide them. **Note: We are not going to look at division.**

Addition with Complex Numbers

Example 1: $(3 + 4i) + (7 - 20i)$

$3+4i+7-20i$

$10-16i$

You try: $(6- \ i) + (3 - 2i)$

$9-3i$

Subtraction with Complex Numbers

Example 2: $(3 + 4i) - (7 - 20i)$

$3+4i-7+20i$

$-4+24i$

You try: $(6 - i) - (3 - 2i)$

$6-i-3+2i$

$3+i$

Multiplication with Complex Numbers (Note: rewrite i^2 as -1)

Example 3: $(1 + 3i)(4 - 2i)$

$4 - 2i + 12i \cancel{-6i^2}$ $+6$

$10 + 10i$

You try: $(6 - i)(3 - 2i)$

$18 - 12i - 3i \cancel{+2i^2}$ -2

$16 - 15i$

Multiply the following complex numbers with its conjugate:

$x+i$, $x-i$ are conjugates

1. $(x + i)(x - i) =$ $x^2 \cancel{-xi} \cancel{+xi} \cancel{-i^2}$ $+1$ $= x^2 + 1$

2. $(x + 5i)(x - 5i) =$ $x^2 \cancel{-5xi} \cancel{+5xi} \cancel{-25i^2}$ $+25$ $= x^2 + 25$

3. $(5 + 4i)(5 - 4i) =$ $25 \cancel{-20i} \cancel{+20i} \cancel{-16i^2}$ $+16$ $= 41$

What patterns do you notice?

When you multiply a complex # by it's conjugate, you always get a real number.

Show that for any real numbers a and b, $(a + bi)(a - bi)$ is a real number.

$a^2 \cancel{-abi} \cancel{+abi} \cancel{-b^2 i^2}$ 0 $+b^2$

$a^2 + b^2$ ⇐ Real # (No imag. part)

The product of a Complex # and its conjugate is a polynomial with real coefficients.

Example 4: How would you verify that $-1 \overset{x_1}{+} 2i$ and $-1 \overset{x_2}{-} 2i$ are solutions to $x^2 + 2x + 5 = 0$? Go ahead and see.

ck: $(-1+2i)^2 + 2(-1+2i) + 5 = 0$

$(-1+2i)(-1+2i) - 2 + 4i + 5 = 0$

$1 - 2i - 2i + \underbrace{\cancel{4i^2}}_{-4} - 2 + 4i + 5 = 0$

$0 = 0$ ✓

$x = -1-2i$: $(-1-2i)^2 + 2(-1-2i) + 5 = 0$

$(-1-2i)(-1-2i) - 2 - 4i + 5 = 0$

$1 + \cancel{2i} + \cancel{2i} + \underbrace{\cancel{4i^2}}_{-4} - 2 - \cancel{4i} + 5 = 0$

$-5 + 5 = 0$

$0 = 0$ ✓

You do:

Express the quantities below in a + bi form, then graph and label the corresponding points on the complex plane.

1. $(1 + i) - (1 - i)$

$1 + i - 1 + i$

$2i$

2. $(1 + i)(1 - i)$

$1 \cancel{- i + i} - i^2$

$0 + 1$

2

3. $i(2 - i)(1 + 2i)$

$(2i - i^2)(1 + 2i)$

$+1$

$(2i + 1)(1 + 2i)$

$2i + 4i^2 + 1 + 2i$

-4

$-3 + 4i$