

4-2 HW Answer Key

1. $3 + 5i$
2. Yes, real numbers can be written as $a + bi$ where $b = 0$. (Ex. $3 + 0i$)
3. 24
4. -12
5. $6i\sqrt{3}$
6. $20 - i$
7. $13 - 9i$
8. $-5 + 31i$
9. $3 - 3i$
10. $22 + 14i$
11. $16 - 38i$
12. $3 - 2i, -4 + 6i, x + 5i$
13. $x^2 + 49$
14. 34

See attached for graph key

Name Key

Alg 2 Homework 4-2

1. Give an example of a complex number. $3+5i$ 2. Are real numbers complex? YesExplain. Real #'s can be written as $a+bi$ where $b=0$.
Ex. $3 = 3+0i$ Simplify:

3. $(2\sqrt{6})^2 = 4(6) = 24$

4. $(\sqrt{-6})(\sqrt{-24}) = i\sqrt{6} \cdot i\sqrt{24} = i^2\sqrt{144} = -1(12) = -12$

5. $(\sqrt{6})(\sqrt{-18}) = \sqrt{6} \cdot i\sqrt{18} = i\sqrt{108} = i\sqrt{36}\sqrt{3} = 6i\sqrt{3}$

Add, subtract or multiply the following complex numbers.

6. $(13 + 4i) + (7 - 5i) = 20 - i$

7. $(8 - 6i) + (5 - 3i) = 13 - 9i$

8. $(3 + 11i) - (8 - 20i) = -5 + 31i$

9. $(8 - 6i) - (5 - 3i) = 3 - 3i$

$$\begin{aligned} 10. (2 + 4i)(5 - 3i) &= 10 - 6i + 20i - 12i^2 & 11. (7 - i)(3 - 5i) &= 21 - 35i - 3i + 5i^2 \\ &= 10 + 14i + 12 & &= 21 - 38i - 5 \\ &= 22 + 14i & &= 16 - 38i \end{aligned}$$

(Continued on back→)

12. What are the conjugates of:

$$(3 + 2i) \text{? } 3 - 2i$$

$$(4 - 6i) \text{? } -4 + 6i$$

$$(x - 5i) \text{? } x + 5i$$

Multiply the following complex numbers with its conjugate:

$$\begin{aligned} 13. (x + 7i)(x - 7i) &= x^2 - 7xi + 7xi - 49i^2 \\ &= x^2 + 49 \end{aligned}$$

$$\begin{aligned} 14. (3 + 5i)(3 - 5i) &= 9 - 15i + 15i - 25i^2 \\ &= 9 + 25 \\ &= 34 \end{aligned}$$

Express the quantities below in a + bi form, then graph and label the corresponding points on the complex plane.

$$1. (-3 + 2i) + (-3 - 2i) = -6 + 0i$$

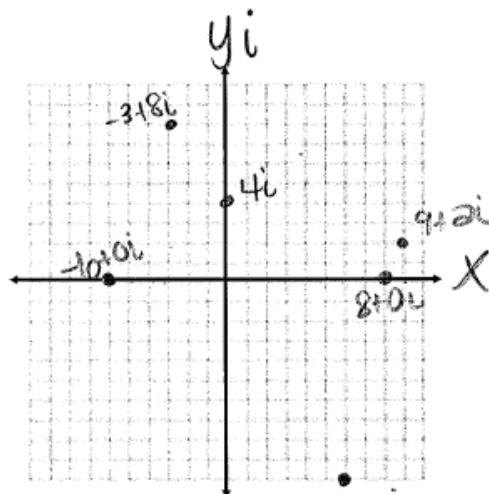
$$2. (3 + 2i) - (3 - 2i) = 4i$$

$$3. (4 + 5i) - (7 - 3i) = -3 + 8i$$

$$4. (4 + 5i) + (5 - 3i) = 9 + 2i$$

$$\begin{aligned} 5. (2 + 2i)(2 - 2i) &= 4 - 4i + 4i - 4i^2 \\ &= 4 + 4 \\ &= 8 + 0i \end{aligned}$$

$$\begin{aligned} 6. (-4 + i)(-2 + 2i) &= 8 - 8i - 2i + 2i^2 \\ &= 8 - 10i - 2 \\ &= 6 - 10i \end{aligned}$$



Powers of i and more operations with complex numbers

$$i^1 = i \quad i^2 = -1$$

Since $i^2 = -1$, we can see that:

$$i^3 = i^1 \cdot i^2 = i(-1) = -i$$

$$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$$

Plot i, i^2, i^3 and i^4 on the complex plane to the right.

Simplify the following:

$$i^5 = i^4 \cdot i^1 = i$$

$$i^6 = i^4 \cdot i^2 = -1$$

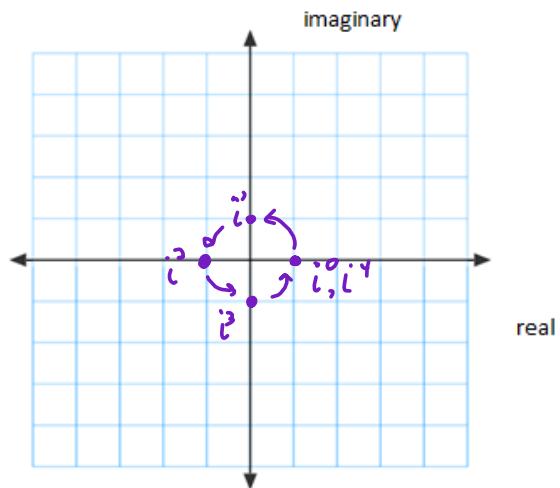
$$i^7 = i^4 \cdot i^3 = -i$$

$$i^8 = i^4 \cdot i^4 = 1$$

$$i^9 = i^4 \cdot i^4 \cdot i^1 = i$$

Powers of i

| |
|------------|
| $i^0 = 1$ |
| $i^1 = i$ |
| $i^2 = -1$ |
| $i^3 = -i$ |
| $i^4 = 1$ |



Simplify i^{102} . Divide exponent by 4

$$i^{102} = (i^4)^{25} \cdot (i)^2 = i^2 = -1$$

$$\sqrt[4]{102} = R_1 \quad .25 = R_2 \quad .5 = R_3$$

$$\begin{array}{r} 25 \\ \sqrt[4]{102} \\ \hline -8 \\ \overline{-22} \\ -20 \\ \hline 2 \end{array}$$

Simplify the following:

$$\begin{aligned} 1. i^{26} &= \cancel{(i^4)^6} \cdot i^{-1} = -1 \\ 26/4 &= 6R2 \\ 2. i^{43} &= \cancel{i^3} = -i \\ 43/4 &= 10R3 \\ 3. i^{60} &= \cancel{i^0} = 1 \\ 60/4 &= 15R0 \end{aligned}$$

$$\begin{aligned} 4. i^{421} &= \cancel{i^1} = i \\ 421/4 &= 105R1 \\ 5. (i^6)(i^7) &= \cancel{i^{13}} = \cancel{i^1} = i \\ 13/4 &= 3R1 \\ 6. 3i^2 - 6i^{12} &= \\ 3(-1) - 6(\cancel{i^0}) &= \\ -3 - 6 &= \textcircled{-9} \end{aligned}$$

More practice simplifying.

$$\begin{aligned} 7. \sqrt{-4} - \sqrt{-100} &= \\ i\sqrt{4} - i\sqrt{100} &= \\ 2i - 10i &= \textcircled{-8i} \end{aligned}$$

$$\begin{aligned} 8. 2\sqrt{-25} - 3\sqrt{-49} &= 2(5i) - 3(7i) \\ &= 10i - 21i \\ &= \textcircled{-11i} \end{aligned}$$

$$\begin{aligned} 9. 2\sqrt{-50} + \sqrt{-32} &= \\ 2i\sqrt{50} + i\sqrt{32} &= \\ 2i\cancel{\sqrt{5}\sqrt{2}} + i\sqrt{16}\sqrt{2} &= \\ 10i\sqrt{2} + 4i\sqrt{2} &= \\ 14i\sqrt{2} & \end{aligned}$$

$$\begin{aligned} 10. \sqrt{-81} \cdot \sqrt{-25} &= \\ (\cancel{9i} \cdot i\sqrt{25}) &= \\ 9i \cdot 5i &= 45i^2 = \textcircled{-45} \end{aligned}$$

$$\begin{aligned} 11. \sqrt{-5} \cdot \sqrt{-80} &= \\ (\cancel{i}\sqrt{5} \cdot \cancel{i}\sqrt{80}) &= \\ \cancel{i^2} \cancel{\sqrt{400}} &= \textcircled{-20} \end{aligned}$$

$$\begin{aligned} 12. -3\sqrt{-10} \cdot \sqrt{-10} &= -3(-10) \\ -3i\sqrt{10} \cdot i\sqrt{10} &= \textcircled{30} \\ -3\cancel{i^2}\cancel{\sqrt{100}} & \\ 30 & \end{aligned}$$

Express each of the following in $a + bi$ form.

13. $(2 + 5i) + (4 + 3i) =$

$$(6 + 8i)$$

14. $(-1 + 2i) - (4 - 3i) =$

$$-5 + 5i$$

$$-1 + 2i - 4 + 3i$$

15. $(4 + i) + (2 - i) - (1 - i) =$

$$4+i+2-i-1+i = 5+i$$

16. $(5 + 2i)(5 - 3i) =$

$$\begin{aligned} & \text{mult!} & 25 - 15i + 15i - 9i^2 \\ & & = 25 + 9 \\ & & = 34 \end{aligned}$$



Find the real values of x and y using the fact that if $a + bi = c + di$, then $\underline{a} = \underline{c}$ and $\frac{\underline{b}}{\underline{bi}} = \underline{d}$.

17. $5x + 3yi = 20 + 9i$

| | |
|-------------|-----------------------------------|
| <u>Real</u> | <u>Imag.</u> |
| $5x = 20$ | $3yi = 9i$ |
| $x = 4$ | $\frac{3i}{3i} y = \frac{9i}{3i}$ |
| | $y = 3$ |

18. $-4 + yi = -12x - i + 8$

| | |
|-----------------|--------------------------------|
| <u>Real</u> | <u>Imag.</u> |
| $-4 = -12x + 8$ | $yi = -i$ |
| -8 | $\frac{i}{i} y = \frac{-i}{i}$ |
| $-12 = -12x$ | $y = -1$ |
| $1 = x$ | |

Complex numbers as solutions to equations

Algebra 2 Unit 4 Day 4

Today, we are going to use the quadratic formula to solve quadratic equations.

Recall the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ The discriminant is the number under the radical or $b^2 - 4ac$.Working with your partner, determine the discriminant and then solve the following quadratic equations using the quadratic formula.

$$\begin{array}{lll}
\begin{aligned} & a=1, b=0, c=-9 \\
1. & x^2 - 9 = 0 & 2. x^2 - 6x + 9 = 0 & 3. x^2 + 9 = 0 \\
& b^2 - 4ac = 0 - 4(1)(-9) & b^2 - 4ac = 36 - 4(1)(9) & b^2 - 4ac = 0 - 4(1)(9) \\
& = 36 & = 0 & = -36 \\
& x = \frac{0 \pm \sqrt{36}}{2(1)} = \frac{\pm 6}{2} & x = \frac{6 \pm \sqrt{0}}{2(1)} = 0 & x = \frac{0 \pm \sqrt{-36}}{2(1)} = \pm \frac{6i}{2} = \pm 3i \\
& x = \pm 3 & &
\end{aligned}
\end{array}$$

How does the value of the discriminant relate to the solutions you found?