

4-3 HW Answer Key

$i^0 = \underline{\text{1}}$

$i^1 = \underline{i}$

$i^2 = \underline{-1}$

$i^3 = \underline{-i}$

$i^4 = \underline{\text{1}}$

$i^5 = \underline{i}$

$i^6 = \underline{-1}$

$i^7 = \underline{-i}$

$i^8 = \underline{\text{1}}$

$i^9 = \underline{i}$

$i^{10} = \underline{-1}$

$i^{11} = \underline{-i}$

1. i

6. -1

2. i

7. -5

3. $-i$

8. 1

4. 1

9. -9

5. i

10. $-2i$

11. $11 + 14i$

12. -1

13. $x = 8, y = 3$

14. $x = -3, y = -5$

15. $x = \pm 6i$

16. $x = \pm 2i$

17. $12i\sqrt{2}$

18. See attached for graph.

Name Key

Alg 2 Homework 4-3

Simplify the following:

$i^0 = 1$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^5 = i$

$i^6 = -1$

$i^7 = -i$

$i^8 = 1$

$i^9 = i$

$i^{10} = -1$

$i^{11} = -i$

Simplify the following:

1. $i^{33} = i^{32} \cdot i = i$

2. $i^{425} = i^{424} \cdot i = i$

3. $i^{51} = i^{48} \cdot i^3 = -i$

4. $i^{64} = 1$

5. $-i^{11} = -i^8 \cdot i^3 = i$

6. $i^{18} = i^{16} \cdot i^2 = -1$

7. $5i^{10} = 5i^8 \cdot i^2 = 5(-1) = -5$

8. $(i^5)(i^7) = i^{12} = 1$

9. $4i^2 - 5i^{16} = 4(-1) - 5(1) = -4 - 5 = -9$

10. $2i^{15} = 2i^{12} \cdot i^3 = 2(-i) = -2i$

Express each of the following in a + bi form.

11. $(8 + 9i) + (3 + 5i) = 11 + 14i$

12. $(7 + 2i) + (3 - 5i) - (11 - 3i) =$
 $10 - 3i - 11 + 3i$
 $= -1$

Find the real values of x and y.

13. $3x + 6yi = 24 + 18i$
 $3x = 24 \quad 6y = 18$
 $x = 8 \quad y = 3$

14. $5(x - 1) + 3yi = -15i - 20$
 $5x - 5 = -20 \quad 3y = -15$
 $5x = -15 \quad y = -5$
 $x = -3$

Solve for x and put your answer in a + bi form.

15. $2x^2 + 72 = 0$
 $2x^2 = -72$
 $x^2 = -36$
 $x = \pm 6i$

16. $4x^2 + 16 = 0$
 $4x^2 = -16$
 $x^2 = -4$
 $x = \pm 2i$

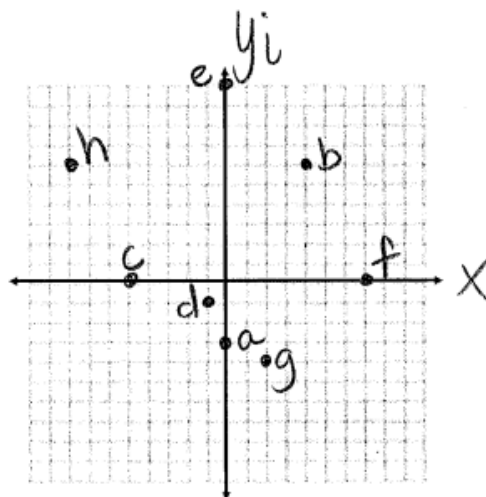
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Simplify.

$$\begin{aligned}
 17. \sqrt{-24} \cdot 2\sqrt{3} &= i\sqrt{24} \cdot 2\sqrt{3} \\
 &= 2i\sqrt{72} \\
 &= 2i\sqrt{36\sqrt{2}} = 12i\sqrt{2}
 \end{aligned}$$

Graph and label the corresponding points on the complex plane.

- a. $-3i$
- b. $4 + 6i$
- c. -5
- d. $-1 - i$
- e. $0 + 10i$
- f. $7 - 0i$
- g. $2 - 4i$
- h. $-8 + 6i$



Complex numbers as solutions to equations

Algebra 2 Unit 4 Day 4

Today, we are going to use the quadratic formula to solve quadratic equations.

Recall the quadratic formula:

The discriminant is the number under the radical or _____.

Working with your partner, **determine the discriminant** and then solve the following quadratic equations **using the quadratic formula**.

1. $x^2 - 9 = 0$

2. $x^2 - 6x + 9 = 0$

3. $x^2 + 9 = 0$

$$\begin{aligned} b^2 - 4ac &= 36 - 4(1)(9) \\ &= 0 \\ x &= \frac{6 \pm \sqrt{0}}{2(1)} \\ &= \frac{6}{2} = 3 \end{aligned}$$

$$\begin{aligned} b^2 - 4ac &= 0 - 4(1)(9) \\ &= -36 \\ x &= \frac{0 \pm \sqrt{-36}}{2(1)} \\ &= \frac{\pm 6i}{2} = \pm 3i \end{aligned}$$

How does the value of the discriminant relate to the solutions you found?

Complex numbers as solutions to equations

Algebra 2 Unit 4 Day 4

Today, we are going to use the quadratic formula to solve quadratic equations.

Recall the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ The discriminant is the number under the radical or $b^2 - 4ac$.Working with your partner, **determine the discriminant** and then solve the following quadratic equations using the quadratic formula.

$a=1 \quad b=0 \quad c=-9$

1. $x^2 - 9 = 0$

$$b^2 - 4ac = 0 - 4(1)(-9)$$

$$= 36$$

$$x = \frac{0 \pm \sqrt{36}}{2(1)} = \frac{\pm 6}{2}$$

$$x = \pm 3$$

2. $x^2 - 6x + 9 = 0$

$$b^2 - 4ac = 36 - 4(1)(9)$$

$$= 0$$

$$x = \frac{6 \pm \sqrt{0}}{2(1)}$$

$$= \frac{6}{2} = 3$$

3. $x^2 + 9 = 0$

$$b^2 - 4ac = 0 - 4(1)(9)$$

$$= -36$$

$$x = \frac{0 \pm \sqrt{-36}}{2(1)}$$

$$= \frac{\pm 6i}{2} = \pm 3i$$

How does the value of the discriminant relate to the solutions you found?

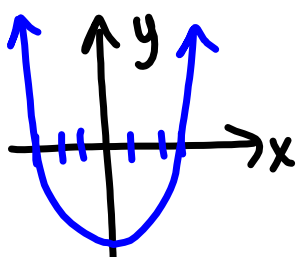
If $b^2 - 4ac < 0$, the solutions will be imaginary (complex)

If $b^2 - 4ac = 0$, we will have 1 Real solution (2 equal)

If $b^2 - 4ac > 0$, the solution will be 2 Real solutions

Using your graphing calculator, sketch a graph of each of the quadratic equations from above.

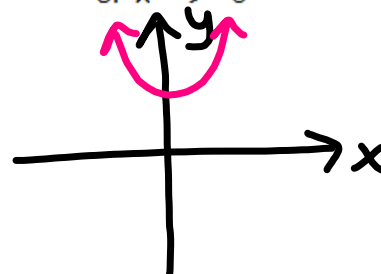
1. $x^2 - 9 = 0$



2. $x^2 - 6x + 9 = 0$



3. $x^2 + 9 = 0$



Conclusions:

② When the graph intersects x-axis only once, then there is 1 Real solution, and the discriminant is zero.

① When the graph intersects x-axis twice, then there are 2 Real solutions, and the discriminant is positive (>0).

When the graph does not intersect x-axis, then there are complex (imag.) roots, and the discriminant is negative (<0).

For 1- 7 do the following:

1. Compute the value of the discriminant of the quadratic equation.
2. Use the value of the discriminant to predict the number and type of solutions.
3. Find all real and complex solutions.

Do first one together:

1. $3x + x^2 = -7 \Rightarrow x^2 + 3x + 7 = 0$

$b^2 - 4ac = 9 - 4(1)(7) = 9 - 28 = -19$ Complex Solutions

$x = \frac{-3 \pm \sqrt{-19}}{2(1)} = \frac{-3 \pm i\sqrt{19}}{2} = \boxed{\frac{-3}{2} \pm \frac{i\sqrt{19}}{2}}$

$a + bi$

$a + bi$ form

Work on the following individually, checking your answers with your partner as you complete each one.

2. $x^2 + 4 = 0$

$b^2 - 4ac = 0 - 4(1)(4)$
 $= -16$

2 Complex solutions

$x = \frac{0 \pm \sqrt{-16}}{2(1)}$

$= \pm \frac{4i}{2}$

$= \pm 2i \quad \{ \pm 2i \}$

$$3. x^2 + 2x + 1 = 0$$

$$b^2 - 4ac = 4 - 4(1)(1) \\ = 0$$

1 real solution

$$x = \frac{-2 \pm \sqrt{0}}{2(1)}$$

$$= \frac{-2}{2}$$

$$= -1 \quad \{-1\}$$

$$4. 3x^2 + 4x + 2 = 0$$

$$b^2 - 4ac = 16 - 4(3)(2) \\ = -8$$

2 complex solutions

$$x = \frac{-4 \pm \sqrt{-8}}{2(3)}$$

$$= \frac{-4 \pm i\sqrt{4}\sqrt{2}}{6}$$

$$= \frac{-4 \pm 2i\sqrt{2}}{6}$$

$$= \frac{-2 \pm i\sqrt{2}}{3} \quad \left\{ -\frac{2}{3} \pm \frac{i\sqrt{2}}{3} \right\}$$

5. $x = 2x^2 + 5$

$$2x^2 - x + 5 = 0$$

$$b^2 - 4ac = 1 - 4(2)(5) \\ = -39$$

2 complex solutions

$$x = \frac{1 \pm \sqrt{-39}}{2(2)}$$

$$= \frac{1 \pm i\sqrt{39}}{4}$$

$$\left\{ \frac{1}{4} \pm \frac{i\sqrt{39}}{4} \right\}$$

6. $9x^2 - 4x - 14 = 0$

$$b^2 - 4ac = 16 - 4(9)(-14) \\ = 520$$

2 real solutions

$$x = \frac{4 \pm \sqrt{520}}{2(9)}$$

$$= \frac{4 \pm \sqrt{4} \sqrt{130}}{18}$$

$$= \frac{4 \pm 2\sqrt{130}}{18}$$

$$= \frac{2 \pm \sqrt{130}}{9}$$

$$\left\{ \frac{2 \pm \sqrt{130}}{9} \right\}$$

7. $8x^2 + 4x + 32 = 0$ (Note: Easier if you factor first)

4

$$2x^2 + x + 8 = 0$$

$$b^2 - 4ac = 1 - 4(2)(8) \\ = -63$$

2 complex solutions

$$x = \frac{-1 \pm \sqrt{-63}}{2(2)}$$

$$= \frac{-1 \pm i\sqrt{9}\sqrt{7}}{4}$$

$$= \frac{-1 \pm 3i\sqrt{7}}{4}$$

$$\left\{ -\frac{1}{4} \pm \frac{3i\sqrt{7}}{4} \right\}$$