1) $(x+4)(x-1)(x+2)(x+1)$
2) $(x-3)(x+2)(x-2)(x+1)$
3) $(x^2+2)(x+5)(x-1)$
4) $(x+3)(x-3)(x^2+x+2)$
5) $(x+2)(x-2)(x^2+2x+4)$
6) $(x^2+2)(x^2+4)(x+2)(x-2)$
7) $x^2+y^2=10$

8) $b=7$
9) 1
10) $h=-2$, $k=5$
11) $h=4$, $k=-2$

Factor each of the following completely.
Hint: you may need to use substitution to make the question look more familiar.
Finish for HW any problems you did not complete in class.

1. $(x^2 + 3x)^2 - 2(x^2 + 3x) - 8$

Let $u = (x^2+3x)$

u^2-2u-8

$(u-4)(u+2)$

$(x^2+3x-4)(x^2+3x+2)$

$=(x+4)(x-1)(x+2)(x+1)$

2. $(x^2 - x)^2 - 8(x^2 - x) + 12$

Let $u = (x^2-x)$

$u^2-8u+12$

$(u-6)(u-2)$

$(x^2-x-6)(x^2-x-2)$

$=(x-3)(x+2)(x-2)(x+1)$

3. $\underbrace{x^4 - 3x^2 - 10}_{} + \underbrace{4x^3 + 8x}_{}$

$(x^2-5)(x^2+2) + 4x(x^2+2)$

$(x^2+2)(x^2+4x-5)$

$= (x^2+2)(x+5)(x-1)$

4. $\underbrace{x^4 - 7x^2 - 18}_{} + \underbrace{x^3 - 9x}_{}$

$(x^2-9)(x^2+2) + x(x^2-9)$

$= (x^2-9)(x^2+x+2)$

$= (x+3)(x-3)(x^2+x+2)$

5. $x^4 + 2x^3 - 8x - 16$

$= x^3(x+2) - 8(x+2)$

$= (x+2)(x^3-8)$

$= (x+2)(x-2)(x^2+2x+4)$

Aside:

$x^3 - 8 = (x-2)(x^2+2x+4)$

$a = x$

$b = 2$

6. $x^6 + 2x^4 - 16x^2 - 32$

$= x^4(x^2+2) - 16(x^2+2)$

$= (x^2+2)(x^4-16)$

$= (x^2+2)(x^2+4)(x^2-4)$

$= (x^2+2)(x^2+4)(x+2)(x-2)$

7. Let x and y be integers such that $x^3 - y^3 = 48$. If $\underline{x - y} = 3$ and $3xy = 18$, what is $x^2 + y^2$?

$a = x$
$b = y$

$x^3 - y^3 = 48$

$xy = 6$

$(x-y)(x^2+xy+y^2) = 48$

$\frac{3(x^2+6+y^2)}{3} = \frac{48}{3}$

$x^2+6+y^2 = 16$

$x^2+y^2 = 10$

8. If $(2x^2 + bx - 10)(x + 5) = 2x^3 + 17x^2 + 25x - 50$ is true for all values of x, what is b?

$x(2x^2+bx-10)+5(2x^2+bx-10) = 2x^3+17x^2+25x-50$

$\boxed{2x^3+bx^2-10x+10x^2+5bx-50=2x^3+17x^2+25x-50}$

$-2x^3 \qquad +50 \quad -2x^3 \qquad +50$

$bx^2+10x^2-10x+5bx = 17x^2+25x$

$-10x^2 \quad +10x \qquad -10x^2+10x$

$bx^2+5bx = 7x^2+35x$

$bx^2 = 7x^2 \qquad 5bx = 35x$

$b=7 \qquad b=7$

$bx^2+10x^2=17x^2$

$b+10=17$

$b=7$

$-10+5b=25$

$5b=35$

$b=7$

9. If $(2x + 3)(4x^2 - 5x + 6) = ax^3 + bx^2 + cx + d$, what is the value of $2b + c$?

$2x(4x^2-5x+6)+3(4x^2-5x+6)$

$8x^3-10x^2+12x+12x^2-15x+18$

$8x^3+2x^2-3x+18$

$a=8 \quad c=-3$

$b=2 \quad d=18$

$2b+c=2(2)+(-3)$

$=1$

10. From Alg2CC Regents January 2017
Algebraically determine the values of h and k to correctly complete the identity stated below:

$$2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k$$

$$2x^3 - 10x^2 + 11x - 7 = x(2x^2 + hx + 3) - 4(2x^2 + hx + 3) + k$$

$$2x^3 - 10x^2 + 11x - 7 = 2x^3 + hx^2 + 3x - 8x^2 - 4hx - 12 + k$$

$$-2x^3 \qquad -2x^3$$

$$-10x^2 + 11x - 7 = hx^2 - 8x^2 - 4hx + 3x - 12 + k$$

$$-10x^2 = hx^2 - 8x^2 \qquad 11x = -4hx + 3x \qquad -7 = -12 + k$$

$$-2x^2 = hx^2 \qquad 8x = -4hx \qquad 5 = k$$

$$-2 = h \qquad -2 \stackrel{\checkmark}{=} h$$

11. Similar question

Algebraically determine the values of h and k to correctly complete the identity stated below:

$$3x^3 - 2x^2 - 13x + 8 = (x - 2)(3x^2 + hx - 5) + k$$

$$3x^3 - 2x^2 - 13x + 8 = x(3x^2 + hx - 5) - 2(3x^2 + hx - 5) + k$$

$$\cancel{3x^3} - 2x^2 - 13x + 8 = \cancel{3x^3} + hx^2 - 5x - 6x^2 - 2hx + 10 + k$$

$-2x^2 = +hx^2 - 6x^2$ $\quad$ $-13x = -5x - 2hx$ $\quad$ $8 = 10 + k$

$4x^2 = hx^2$ $\quad$ $-8x = -2hx$ $\quad$ $-2 = k$

$4 = h$ $\quad$ $4 \overset{\checkmark}{=} h$

End Behavior

Warm-Up: Factor completely.

$x^5 - 4x^3 - x^2 + 4$

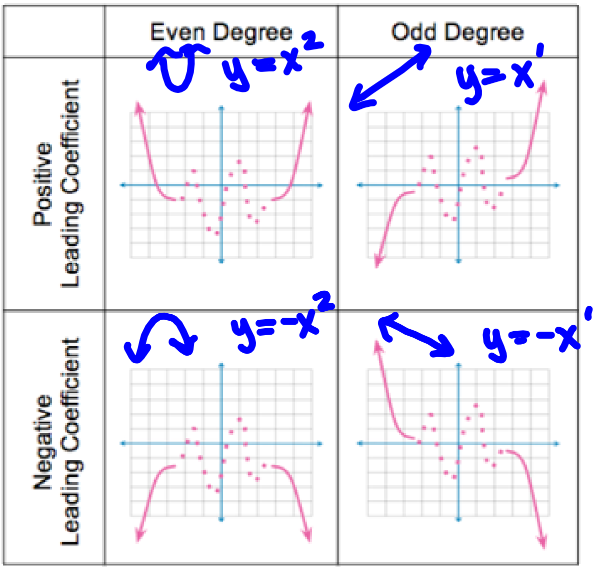
$x^3(x^2-4)-1(x^2-4)$

$(x^2-4)(x^3-1)$

$(x+2)(x-2)(x-1)(x^2+x+1)$

The degree (highest power) and leading coefficient (coefficient of the highest power) of a polynomial determine the **end behavior** for that polynomial.

There are four general shapes for polynomials.



Without your calculator:

a. state degree (highest exponent)

b. state the sign of the leading coefficient

c. sketch (no graph paper) the end behavior

1. $P(x) = 2x^3 - 3x^2 + 4x + 7$

a. 3 (odd)

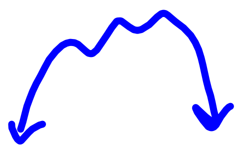
b. +

c. 

2. $P(x) = -4x^8 + 2x - 1$

a. 8 (even)

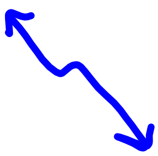
b. −

c. 

3. $P(x) = -6x^{5} + 2x^{2} - 3x$

a. 5 (odd)

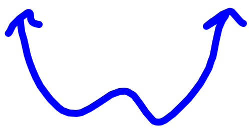
b. $-$

c. 

4. $P(x) = 3x^{4} + 2x^{3} - 4x - 5$

a. 4 (even)

b. $+$

c. 

So, what if our polynomial is in factored form?
How would we find the degree and leading coefficient?

degree - add all of the exponents on variable

L.C - mult. the coeff. on the variables

1. $P(x) = -x^{1}(x^{1} + 2)(x^{1} - 3)$

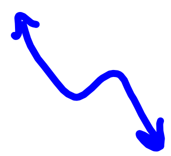
   $1 + 1 + 1$

   $-1(1)(1) = -1$

   a. 3 (odd)

   b. −

   c.



2. $P(x) = -x^{1}(x - 1)^{2}(2x^{1} + 3)$

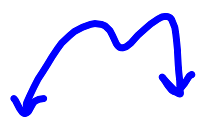
   $1 + 2 + 1 = 4$

   a. 4 (even)

   $-1(1)(2) = -2$

   b. −

   c.



3. $P(x) = x^3(x + 2)(x - 1)$

$3 + 1 + 1 = 5$

a. 5 (odd)

b. +

c.

4. $P(x) = (x - 2)^2(x + 1)$

$2 + 1 = 3$

a. 3 (odd)

b. +

c.

What else do we get from a polynomial in factored form?

Roots / Zeros (x-int.)

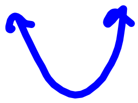
Could we get a better sketch of our polynomial from factored form? How?

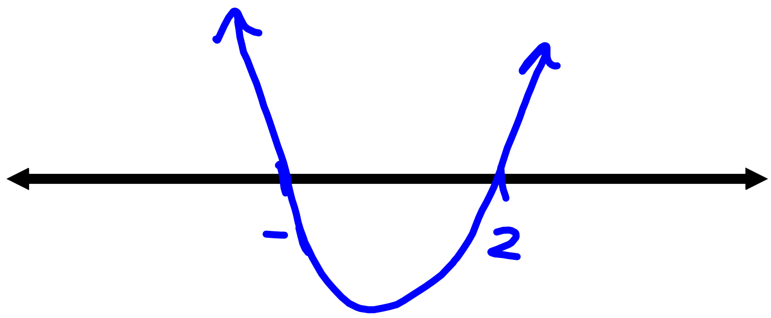
Yes - combine end behavior info. with zeros (x-int.)

How would you sketch $y = (x + 1)(x - 2)$

degree = 2

LC = +





How about $y = (x + 1)^2$

Degree = 2

LC = +