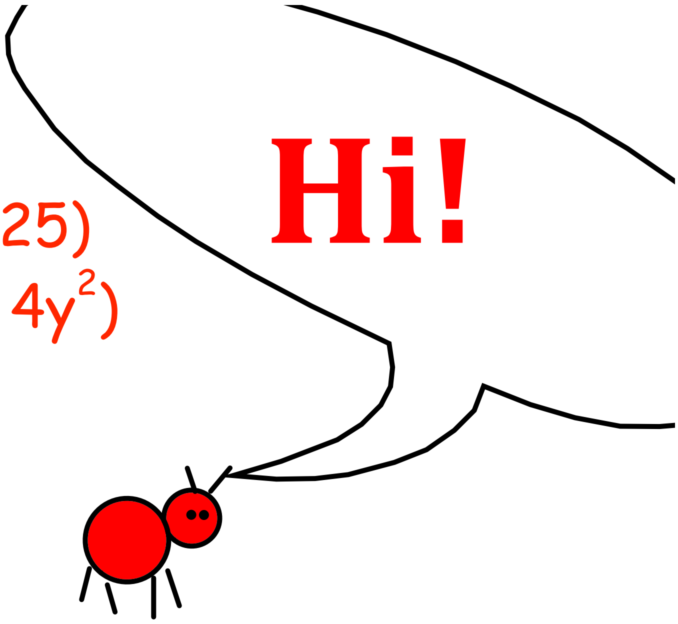
HW 5 - 6

1. $4(x - 1)^2(x + 1)^2$
2. $5x^2(x - 5)(x^2 + 5x + 25)$
3. $(x - 2y)(x + 2y)(x^2 + 4y^2)$
4. $(x + y + z)(x + y - z)$
5. $\{\pm 2i\sqrt{2}, \pm 2\sqrt{2}\}$
6. $\{-2, \pm 4\}$
7. $\{0, \pm 1, 2\}$
8. $\{\pm i\sqrt{2}, \pm 1\}$



In 1 - 4, Factor Completely.

1. $4x^4 - 8x^2 + 4$

$4(x^4 - 2x^2 + 1)$

$= 4(x^2 - 1)(x^2 - 1)$

$= 4(x-1)(x+1)(x-1)(x+1)$

$= 4(x-1)^2(x+1)^2$

2. $5x^5 - 625x^2$

$5x^2(x^3 - 125)$

$= 5x^2(x-5)(x^2+5x+25)$

3. $x^4 - 16y^4$

$(x^2 - 4y^2)(x^2 + 4y^2)$

$= (x-2y)(x+2y)(x^2+4y^2)$

4. $(x + y)^2 - z^2$

Let $u = x+y$

$u^2 - z^2 = (u+z)(u-z)$

$(x+y+z)(x+y-z)$

In 5 - 8, write in factored form and find the zeros.

5. $f(x) = x^4 - 64$

$f(x) = (x^2+8)(x^2-8)$

$0 = (x^2+8)(x^2-8)$

$\sqrt{x^2} = \sqrt{-8}$ $\sqrt{x^2} = \sqrt{8}$

$x = \pm i\sqrt{4}\sqrt{2}$ $x = \pm\sqrt{4}\sqrt{2}$

$x = \pm 2i\sqrt{2}$ $x = \pm 2\sqrt{2}$

$\{\pm 2i\sqrt{2}, \pm 2\sqrt{2}\}$

6. $f(x) = x^3 + 2x^2 - 16x - 32$

$f(x) = x^2(x+2) - 16(x+2)$

$f(x) = (x+2)(x^2-16)$

$f(x) = (x+2)(x-4)(x+4)$

$0 = (x+2)(x-4)(x+4)$

$x = -2,\ x = 4,\ x = -4$

$\{-2, \pm 4\}$

7. $f(x) = x^4 - 2x^3 - x^2 + 2x$

$f(x) = x^3(x-2) - x(x-2)$

$f(x) = (x^3 - x)(x-2)$

$f(x) = x(x^2-1)(x-2)$

$f(x) = x(x-1)(x+1)(x-2)$

$0 = x(x-1)(x+1)(x-2)$

$x = 0,\ x = 1,\ x = -1,\ x = 2$

$\{0, \pm 1, 2\}$

8. $f(x) = x^4 + x^2 - 2$

$f(x) = (x^2+2)(x^2-1)$

$f(x) = (x^2+2)(x+1)(x-1)$

$0 = (x^2+2)(x+1)(x-1)$

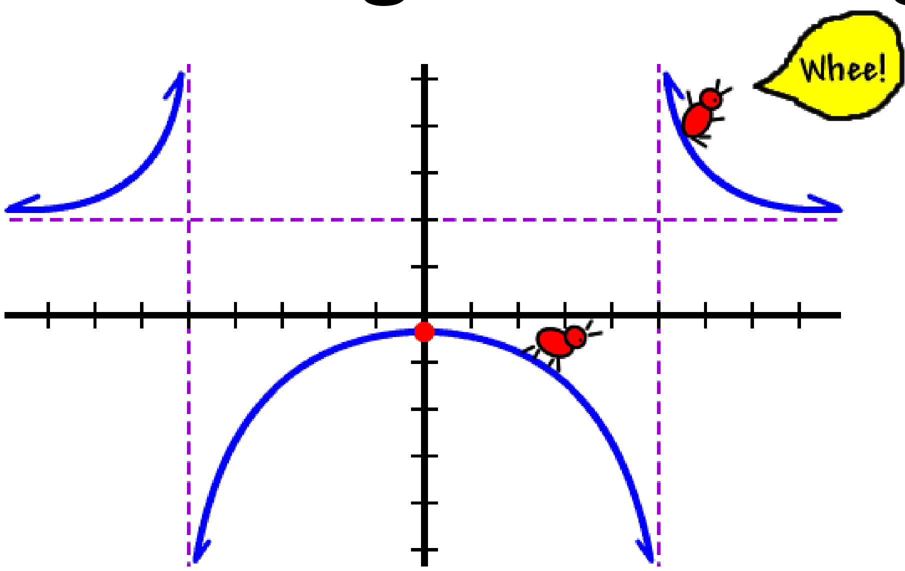
$x^2+2=0$ $x+1=0$ $x-1=0$

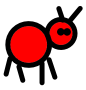
$\sqrt{x^2} = \sqrt{-2}$

$x = \pm i\sqrt{2}$ $x = -1$ $x = 1$

$\{\pm i\sqrt{2}, -1, 1\}$

Increasing/Decreasing





Explain how you would sketch $P(x) = x^2(x - 1)^3(x + 1)$ without a graphing calculator.

Sketch a graph that has 2 real zeros and 2 imaginary zeros.

What do you think it means if a function is increasing? Decreasing?

Interval Notation A **notation** for representing an **interval** as a pair of numbers. The numbers are the endpoints of the **interval**. Parentheses and/or brackets are used to show whether the endpoints are excluded or included. For example, [2, 7) is the **interval** of real numbers between 2 and 7, including 2 and excluding 7.

[2 , 7)

Graphically →

Increasing → a function f is <u>increasing</u> on an interval if, for any 2 points in the interval, a positive change in x results in a positive change for f(x).

Decreasing → a function f is <u>decreasing</u> on an interval if, for any 2 points in the interval, a positive change in x results in a negative change for f(x).

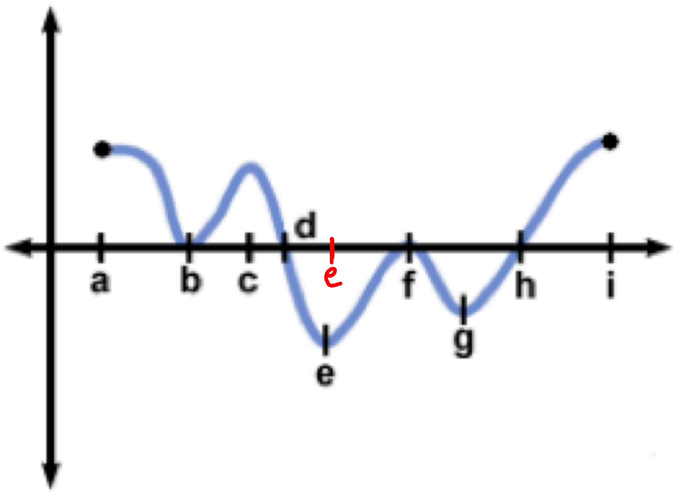
* When determining increasing/decreasing we are concerned with the X - VALUES!!!
And all intervals are written in (,) form

* When determining increasing/decreasing we are concerned with the X - VALUES!!!

Where is the graph at right increasing/decreasing?

Increasing:
(b,c),(e,f),(g,i)

Decreasing?
(a,b),(c,e),(f,g)



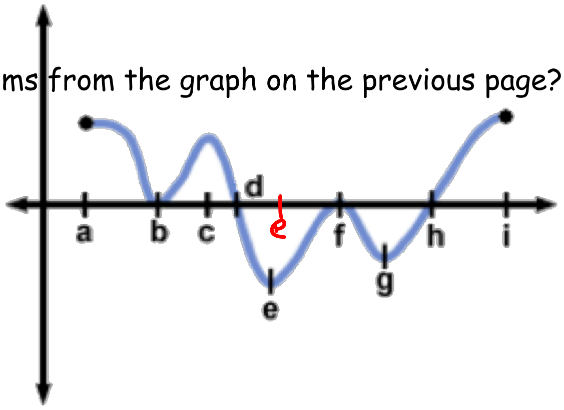
Relative Maximum ⟶ of a function f is a value f(c) that is > all range values of f on some interval containing c.

Relative Minimum ⟶ of a function f is a value f(c) that is < all range values of f on some interval containing c.

Where are the relative minimums and maximums from the graph on the previous page? (shown again here)

Mimimums: b, e, g

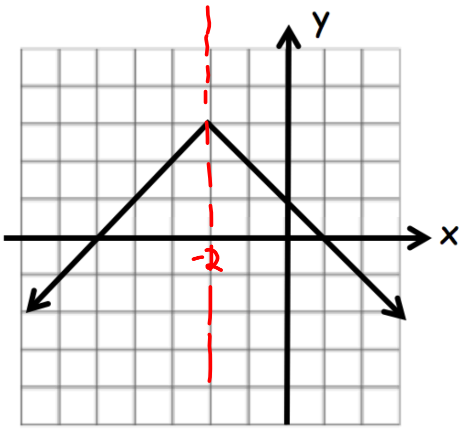
Maximums: a, c, i, f



For each of the following, determine the intervals on which the graph is increasing and decreasing.
Find all relative minima and maxima.

* When determining increasing/decreasing we are concerned with the X - VALUES!!!

1.



Increasing: $(-\infty, -2)$

Decreasing: $(-2, \infty)$

Rel Min: none

Rel Max: $(-2, 3)$

Describe the behavior of the above functions as x approaches positive and negative infinity

$x \to \infty$ The graph is decreasing $y \to -\infty$

$x \to -\infty$ $y \to -\infty$