HW Answers 5-~~6~~ 7

1. $(x^n - 4)(x^n - 1)$

2. $3x(x - 1)(x^2 + x + 1)$

3. $x(2x - 5)$

4. $\{0, 1, -1, 4\}$

5. $\cancel{\{\pm i\sqrt{6}, \pm\sqrt{3}\}}$ $\{\pm 1/2, \pm\sqrt{3}\}$

6. $\{0, 7/5, -7/5\}$

In 1 – 3, Factor Completely; 4 – 6, write in factored form and find the roots.

1. $x^{2n} - 5x^n + 4$

2. $3x^4 - 3x$

3. $2(x - 1)^2 - (x - 1) - 3$ Let $u = x - 1$

$2u^2 - u - 3$

$(2u - 3)(u + 1)$

$(2(x-1) - 3)(x - 1 + 1)$

$2x - 2$

$(2x - 5)(x)$

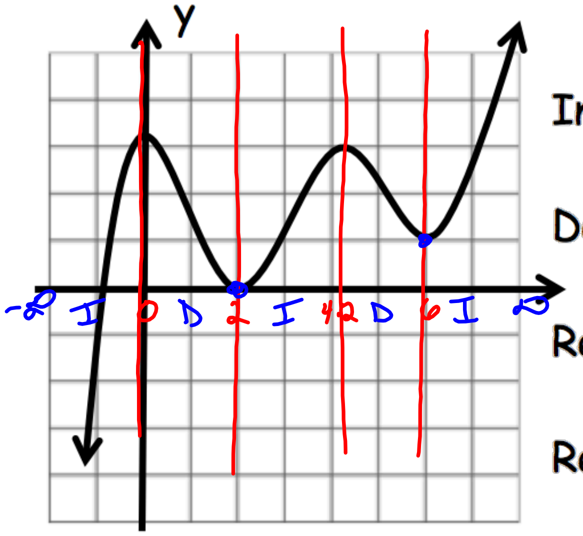
4. $2x^4 + 8x^3 = 2x^2 + 8x$

5. $\cancel{x^4 + 3x^2 - 18 = 0}$

$4x^4 - 13x^2 + 3 = 0$

6. $25x^3 = 49x$

2.



Increasing: $(-\infty, 0), (2, 4.2), (6, \infty)$

Decreasing: $(0, 2), (4.2, 6)$

Rel Min: $(2, 0), (6, 1)$

Rel Max: $(0, 3.2), (4.2, 3)$

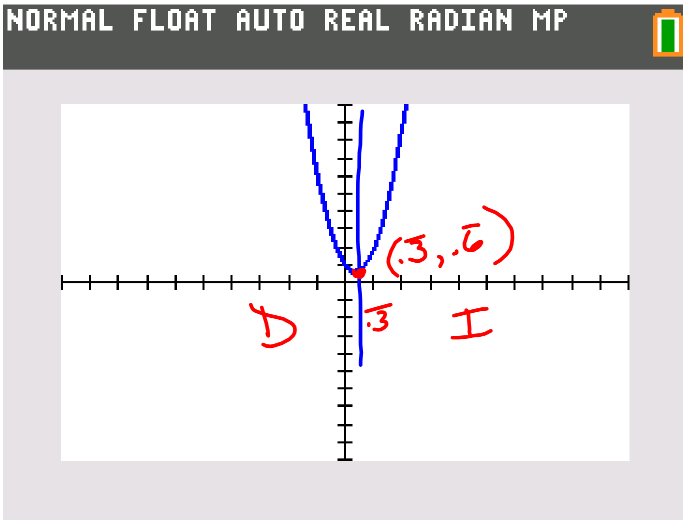
Describe the behavior of the above functions as x approaches positive and negative infinity

$x \to \infty$ $y \to \infty$

$x \to -\infty$ $y \to -\infty$

Using your graphing calculator, sketch each of the following. Determine intervals where increasing, decreasing and any relative minima or maxima.

1. $y = 3x^2 - 2x + 1$



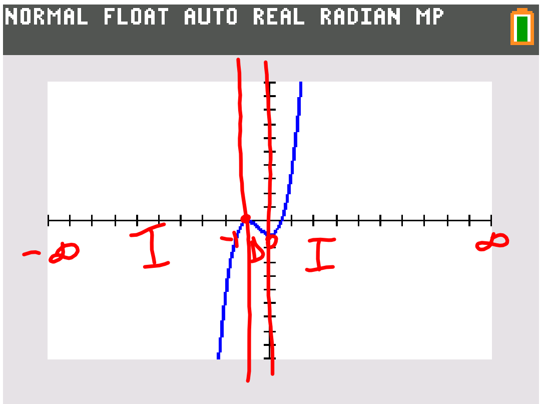
Increasing: $(.\overline{3}, \infty)$

Decreasing: $(-\infty, .\overline{3})$

Rel Min: $(.\overline{3}, .\overline{6})$

Rel Max: None

2. $y = 2x^3 + 3x^2 - 1$



Increasing: $(-\infty, -1), (0, \infty)$

Decreasing: $(-1, 0)$

Rel Min: $(0, -1)$

Rel Max: $(-1, 0)$

Warm-Up: Each of you will be assigned one of these three problems.

Remember: $F(x) \div G(x) = Q(x)$ with a remainder of $R(x)$

work space below - use for your problem...

Which is easier to read as:

$$\text{Dividend} \rightarrow F(x) = \underbrace{G(x)}_{\text{Divisor}} \cdot \underbrace{Q(x)}_{\text{Quotient}} + \underbrace{R(x)}_{\text{Remainder}}$$

1. Consider the polynomial function $f(x) = 3x^2 + 8x - 4$

a. Using long division, divide $f(x)$ by $x - 2$

$Q(x) = 3x+14$

$R(x) = 24$

So $f(x) = (x-2)(3x+14)+24$

$$
\begin{array}{r|l}
 & 3x + 14 \\
x-2 & 3x^2 + 8x - 4 \\
 & -3x^2 + 6x \\
 & \quad 14x - 4 \\
 & \quad -14x + 28 \\
 & \qquad 24
\end{array}
$$

b. Find $f(2)$

$f(2) = 3(2)^2 + 8(2) - 4$
$= 12 + 16 - 4$
$= 24$

Aside:
$3x(x-2) = 3x^2 - 6x$
$14(x-2) = 14x - 28$

2. Consider the polynomial function $g(x) = x^3 - 3x^2 + 6x + 8$

a. Using long division, divide $g(x)$ by $x + 1$

$Q(x) = x^2 - 4x + 10$

$R(x) = -2$

So $g(x) = (x+1)(x^2-4x+10) - 2$

$$
\begin{array}{r}
x^2 - 4x + 10 \\
x+1 \overline{) x^3 - 3x^2 + 6x + 8} \\
-x^3 - x^2 \\
\hline
-4x^2 + 6x \\
4x^2 + 4x \\
\hline
10x + 8 \\
-10x - 10 \\
\hline
-2
\end{array}
$$

Aside:

$x^2(x+1) = x^3 + x^2$

$-4x(x+1) = -4x^2 - 4x$

$10(x+1) = 10x + 10$

b. Find $g(-1)$

$g(-1) = (-1)^3 - 3(-1)^2 + 6(-1) + 8$

$g(-1) = -1 - 3 - 6 + 8$

$g(-1) = -2$

3. Consider the polynomial function $h(x) = x^3 + 0x^2 + 2x - 3$

a. Using long division, divide $h(x)$ by $x - 3$

$Q(x) = x^2+3x+11$

$R(x) = 30$

So $h(x) = (x-3)(x^2+3x+11)+30$

$$\begin{array}{r} x^2+3x+11 \\ x-3\overline{)x^3+0x^2+2x-3} \\ -x^3+3x^2 \\ \hline 3x^2+2x \\ -3x^2+9x \\ \hline 11x-3 \\ -11x+33 \\ \hline 30 \end{array}$$

b. Find $h(3)$

$h(3)=(3)^3+2(3)-3$

$h(3) = 27+6-3$

$h(3) = 30$

Aside:

$x^2(x-3) = x^3-3x^2$

$3x(x-3) = 3x^2-9x$

$11(x-3) = 11x-33$

Write in the answers for all parts, gathered from the class discussion.
What pattern do you see?

Div. by $(x-2)$ $R = 24$ $f(2) = 24$

$(x+1)$ $R = -2$ $f(-1) = -2$

$(x-3)$ $R = 30$ $f(3) = 30$

What can we say about the connection between dividing a polynomial, P, by x – a and the value of P(a)?

Write in the answers for all parts, gathered from the class discussion.
What pattern do you see? (Answers will be posted - get them from your teachers website)

The remainder is the same as the function value of the possible zero.

Look at #1: Possible zero: 2

$$R(2) = f(2)$$

What can we say about the connection between dividing a polynomial, P, by x – a and the value of P(a)?

$$\text{Remainder} = P(a)$$

$$R(x) = P(a)$$

In algebra, the **remainder theorem** is an application of polynomial long division.

The remainder of a polynomial p(x) divided by a linear divisor (x – c) is equal to p(c).

What does that mean?

If you divide a polynomial P(x) by a possible factor (x - c), you will get a remainder that is equal to the function value of the corresponding possible zero

$$\boxed{\text{Formally: } P(x) = q(x)(x - a) + P(a)}$$

Why Is This Useful?

Knowing that x - c is a factor is the same as knowing that c is a root (and vice versa).

The **factor** "x – c" and the **root** "c" are the same thing!

Now try these: Use the remainder theorem to determine the remainder.

1. $(-x^3 + 6x - 7) \div (x - 2)$

$P(2) = -2^3 + 6(2) - 7 = -3$

$R(x) = -3$ (Remaider $= -3$)

2. $(x^3 + x^2 - 5x - 6) \div (x + 2)$

$P(-2) = (-2)^3 + (-2)^2 - 5(-2) - 6$

$P(-2) = -8 + 4 + 10 - 6$

$P(-2) = 0$

$\therefore R(x) = 0$

What do you think it means if the remainder is 0?

The divisor is a factor of the polynomial; the corresponding x-value is a zero of the polynomial.