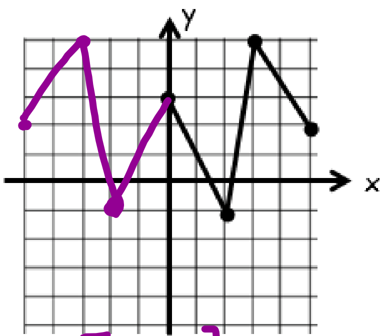
Given the **partial graph** of function f shown below. Sketch the other half of the function in a) if f(x) is **even** and in b) if f(x) is **odd**. Find f(-x) for each of the indicated points. State the domain and range of each completed function.

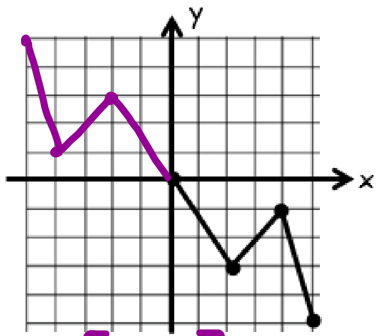
1. Even



Domain: [-5,5]

Range: [-1,5]

2. Odd

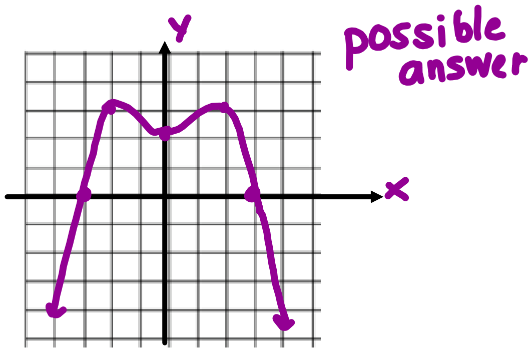


Domain: [-5,5]

Range: [-5,5]

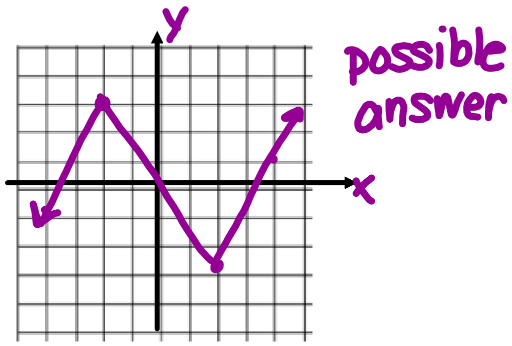
3. Draw a function with the following properties:

   a. Zeros of 3 and -3
   b. As $x \rightarrow \infty$ $y \rightarrow -\infty$
   c. The graph is even



4. Draw a function with the following properties:

   a. It passes through the point (-2, 3)
   b. As $x \rightarrow -\infty$ $y \rightarrow -\infty$
   c. The graph is odd



Given the partially filled out table below for f(x), fill in the rest of the table based on the function type.

7. Even

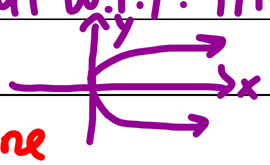
x	-3	-2	-1	0	1	2	3
y	17	7	1	-1	1	7	17

8. Odd

x	-3	-2	-1	0	1	2	3
y	18	2	-2	0	2	-2	-18

9. Even functions have symmetry across the y-axis. Odd functions have symmetry across the origin. Can a function have symmetry across the x-axis? Why or why not?

If a graph is symmetrical w.r.t. the x-axis it's not a funtion. i.e.



b/c fails vert. line test

Determine algebraically whether each of the following functions is even, odd, or neither.
Check your answer using tables on your graphing calculator.

10. $f(x) = 2x + 1$

$f(-x) = 2(-x) + 1$

$= -2x + 1$

not same
not opp
$\therefore$ neither

11. $f(x) = 2x^2 + 1$

$f(-x) = 2(-x)^2 + 1$

$= 2x^2 + 1$

same $\rightarrow$ even

12. $f(x) = 2x^3 + x$

$f(-x) = 2(-x)^3 + (-x)$

$= -2x^3 - x$

opp $\rightarrow$ odd

13. The cubic $x^3 + 7x^2 + 13x + 3$ has only one rational zero, $x = -3$.
Use polynomial long division to show that the remainder is zero when dividing the cubic by $x + 3$.
Then use the quadratic formula to find the other two (irrational) zeros.

$$
\begin{array}{r}
x^2 + 4x + 1 \\
x+3 \,\big)\, \overline{x^3 + 7x^2 + 13x + 3} \\
\underline{-x^3 - 3x^2} \\
4x^2 + 13x \\
-\underline{4x^2 - 12x} \\
x + 3 \\
-\underline{x - 3} \\
0 \; \checkmark
\end{array}
$$

Aside:

$x^2(x+3) = x^3 + 3x^2$

$4x(x+3) = 4x^2 + 12x$

$x^2 + 4x + 1 = 0$

$b^2 - 4ac = 16 - 4(1)(1) = 12$

$\sqrt{12} = 2\sqrt{3}$

$x = \dfrac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3}$

3) $(x+2)(3x^2-1)$

4) $(x^n-2)(x^n+2)(x^n-1)(x^n+1)$

5) $(4x+3)(16x^2-12x+9)$

6) $(x-6)(x-1)$

7) $\{\pm 2, \pm 6\}$

8) $\{3/2, \pm\sqrt{5}\}$

9) $\{0, \pm 3i, \pm 3\}$

10) $\{1/3, 2\}$

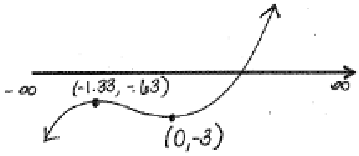
11) $R=-6$
No b/c $R \neq 0$

12) $K=-4$
$(x+2)(x+1)$

Name Key
Date ____________

Alg2CC
Review Unit 5

1. Sketch: $y = 2x^3 + 4x^2 - 3$
Find:
Increasing: $(-\infty, -1.33), (0, \infty)$
Decreasing: $(-1.33, 0)$
Rel Min: $(0, -3)$
Rel Max: $(-1.33, -.63)$



2. Find the zeros of the polynomial, state the multiplicity of each. Sketch (including the end behavior)

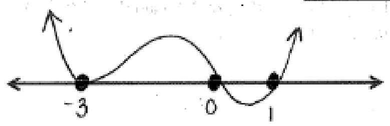
$P(x) = x(x + 3)^2(x - 1)$

Degree: 4

End Behavior: ↑↑

Z	M	T/C
-3	2	T
0	1	C
1	1	C

Sketch:



3 - 6: Factor completely

3. $3x^3 + 6x^2 - x - 2$
$= 3x^2(x+2) - 1(x+2)$
$= (3x^2-1)(x+2)$

4. $x^{4n} - 5x^{2n} + 4$
$= (x^{2n} - 4)(x^{2n} - 1)$
$= (x^n - 2)(x^n + 2)(x^n - 1)(x^n + 1)$

5. $64x^3 + 27 = (4x+3)(16x^2 - 12x + 9)$
$a = 4x$
$b = 3$

6. $(x - 3)^2 - (x - 3) - 6$ Let $u = x - 3$
$u^2 - u - 6$
$(u-3)(u+2)$
$(x-3-3)(x-3+2)$
$(x-6)(x-1)$

7 - 10: Solve (Factor completely first)

7. $x^4 - 40x^2 + 144 = 0$

$(x^2-4)(x^2-36)=0$

$(x-2)(x+2)(x-6)(x+6)=0$

$x=2 \quad x=-2 \quad x=6 \quad x=-6$

$\{\pm 2, \pm 6\}$

8. $2x^3 - 3x^2 - 10x + 15 = 0$

$x^2(2x-3)-5(2x-3)=0$

$(2x-3)(x^2-5)=0$

$2x-3=0 \quad x^2-5=0$

$x=\frac{3}{2} \quad x^2=5$

$x=\pm\sqrt{5}$

$\{\frac{3}{2}, \pm\sqrt{5}\}$

9. $x^5 - 81x = 0$

$\{0, \pm 3i, \pm 3\}$

$x(x^4-9)=0$

$x(x^2+9)(x^2-9)=0$

$x(x^2+9)(x+3)(x-3)=0$

$x=0 \quad x^2+9=0 \quad x+3=0 \quad x-3=0$

$x^2=-9 \quad x=-3 \quad x=3$

$x=\pm 3i$

10. $3(x-1)^2 - (x-1) - 2 = 0$

Let $u=x-1$

$3u^2-u-2=0$

$3u^2-3u+2u-2=0$

$3u(u-1)+2(u-1)=0$

$(3u+2)(u-1)=0$

$(3(x-1)+2)(x-1-1)=0$

$(3x-1)(x-2)=0$

$3x-1=0$

$x=\frac{1}{3}$

$x-2=0$

$x=2$

$\{\frac{1}{3}, 2\}$

11. Use long division to find the quotient (Q(x)) and remainder (R(x)). Verify your remainder with the remainder theorem.

$(2x^3 + 5x^2 + 3x - 4) \div (x + 2)$

$$\begin{array}{r} 2x^2+x+1 \\ x+2\,\overline{)\,2x^3+5x^2+3x-4} \\ -2x^3-4x^2 \quad\quad\quad \\ \hline x^2+3x \quad\; \\ -x^2-2x \quad\; \\ \hline x-4 \\ -x-2 \\ \hline -6 \end{array}$$

Aside:

$2x^2(x+2)=2x^3+4x^2$

$x(x+2)=x^2+2x$

$1(x+2)=x+2$

$\boxed{\begin{array}{l} Q(x)=2x^2+x+1 \\ R(x)=-6 \end{array}}$

$P(-2)=-2(-2)^3+5(-2)^2+3(-2)-4$

$P(-2)=-6$ ✓

Is (x + 2) a factor of $2x^3 + 5x^2 + 3x - 4$? Explain your answer.

No. If x+2 was a factor, the remainder would be 0.

12. Given the polynomial $P(x) = x^3 + x^2 + kx - 4$, find the value of k such that x - 2 is a factor of P.

$P(2)=2^3+2^2+2x-4$

$0=8+2k$

$2k=-8$

$k=-4$

$P(x)=x^3+x^2-4x-4$

Using your value of k, find the other factors of P. (either sketch or algebraically)

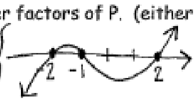
Algebraically

$P(x)=x^2(x+1)-4(x+1)$

$P(x)=(x+1)(x^2-4)$

$P(x)=(x+1)(x-2)(x+2)$

Graphically



$P(x)=(x+2)(x+1)(x-2)$

$\boxed{\begin{array}{l} \therefore \text{ other factors:} \\ (x+1)(x+2) \end{array}}$