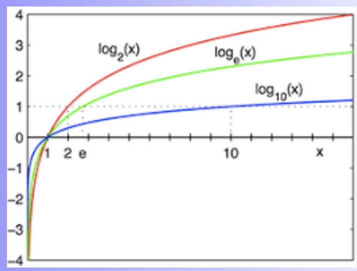
# Introduction to Logarithms

**Index = Logarithm**

$N = a^x$ $\log_a N = x$

Index form Logarithm form




Feb 15-7:51 AM

Introduction to Logarithms Unit 10 Day 1

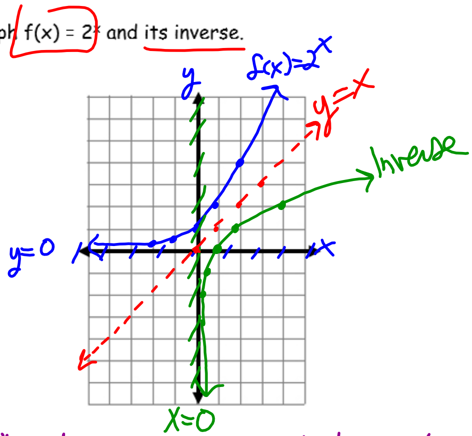
Given the following function, make a table & graph f(x) = $2^x$ and its inverse.

f(x) = $2^x$ → graph

| x | y |
|---|---|
| -2 | 1/4 |
| -1 | 1/2 |
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |

Inverse: → graphed

| x | y |
|---|---|
| 1/4 | -2 |
| 1/2 | -1 |
| 1 | 0 |
| 2 | 1 |
| 4 | 2 |




Is f(x) = $2^x$ a one-to-one function? Explain your answer.

Yes is 1-1. f(x) passes both the vertical and horizontal line tests. Every x maps to one y AND every y maps to one x.

Based on your answer, what must be true about the inverse of this function?

The inverse must also be a function and pass the vertical line test.

Based on the graph of the inverse of f(x) above, ← Inverse

Describe the end behavior of the function as $x \to \infty$ $y \to \infty$

Describe the end behavior of the function as $x \to 0$ $y \to -\infty$

What would be the first two steps to find an equation for the inverse of $\boxed{f(x) = 2^x}$ algebraically?

1. Replace f(x) w/ y

2. Switch x and y

$y = 2^x$

$x = 2^y$

3. Solve for y

$\boxed{y = \log_2 x}$

Inverse

$x = 2 + y \rightarrow x - 2 = y$    +, −

$x = 2y \rightarrow \frac{x}{2} = y$    x, ÷

? exponent, logarithm

To help us with the next step we have to look at a new function called a logarithm.

Defining Logarithmic Equations – The function $\boxed{y = \log_b x}$ is the name we give the inverse of $y = b^x$. For example, $y = \log_2 x$ is the inverse of $y = 2^x$. We can write an **equivalent exponential equation** for each logarithm as follows:

exponent

$$y = \log_b x \text{ is the same as } b^y = x$$

or

$$f^{-1}(x) = \log_b x$$

Now let's write the equation on the previous page as a logarithmic equation.

Logarithmic Equation: $y = \log_2 x$ or $f(x) = \log_2 x$

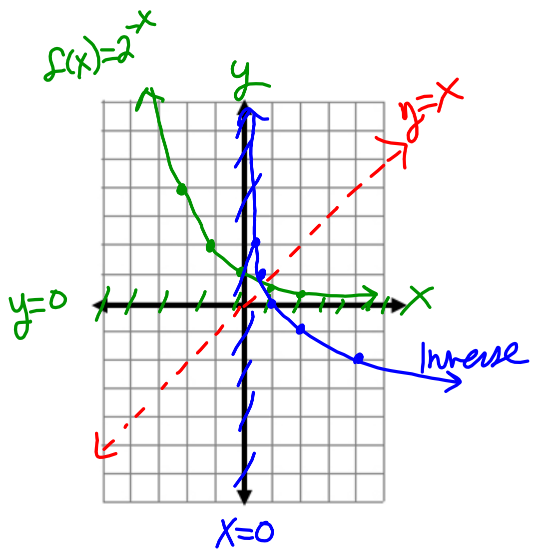
You try:

$f(x) = 2^{-x} = \left(\frac{1}{2}\right)^x$    $y = \frac{1}{2}^x$

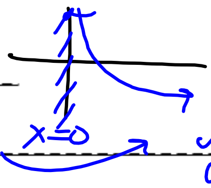
| x | y |
|---|---|
| -2 | 4 |
| -1 | 2 |
| 0 | 1 |
| 1 | 1/2 |
| 2 | 1/4 |

Inverse: $x = \frac{1}{2}^y$

| x | y |
|---|---|
| 4 | -2 |
| 2 | -1 |
| 1 | 0 |
| 1/2 | 1 |
| 1/4 | 2 |



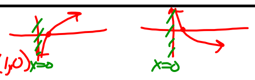

Logarithmic Equation: $y = \log_{\frac{1}{2}} x$



Describe the end behavior of the function as $x \to \infty$   $x \to 0$   $y \to -\infty$

Describe the end behavior of the function as $x \to 0$   $y \to \infty$

Summary




Point on every logarithmic graph: (1,0)

Domain: $\{x \mid x>0\}$, $(0,\infty)$

Quadrants: I, IV

Range: $\{y \mid y \in \mathbb{R}\}$, $(-\infty,\infty)$

Asymptote: $x=0$ (vertical)

Let's graph the function $g(x) = \log_2(x+2)$ left 2

We will use the graph for $f(x) = \log_2 x$ to help us do this.

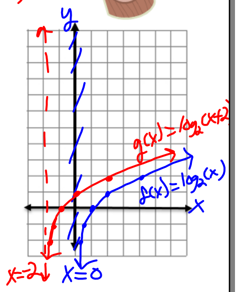
$f(x) = 2^x$

no graph

| x | y |
|---|---|
| -2 | $\frac{1}{4}$ |
| -1 | $\frac{1}{2}$ |
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |

$f(x) = \log_2(x)$ graph

| x | y |
|---|---|
| 1/4 | -2 |
| 1/2 | -1 |
| 1 | 0 |
| 2 | 1 |
| 4 | 2 |




What is the asymptote of $f(x) = \log_2 x$? $x=0$

On the graph, plot the function $g(x) = \log_2(x+2)$ using the points from $f(x) = \log_2 x$

What transformation happened to the function? left 2

What is the asymptote of g(x)? $x=-2$

Describe the end behavior of g(x) as $x \to \infty$ $y \to \infty$

Describe the end behavior of g(x) as $x \to -2$ $y \to -\infty$

So the asymptote for any function $f(x) = \log_b(x-c)$ will be $x=c$

State the transformation and asymptote for the graph $\boxed{f(x) = \log_3(x-4)}$

right 4, $x=4$

Now let's look at the function $g(x) = \log_2(x-3) + 2$.

What transformations happened to this function?

right 3, up 2

What is the asymptote of the new function?

$x=3$