Unit 10 HW 9

Tonight's HW: #1-b-b: Solve Algebraically

1. $\{.9730\}$
2. $\{.2197\}$
3. $t = 31.4$ years
4. (a) $k = .0193$
   (b) $t = 10.4$ years
5. (a) \$1001.86
   (b) 15 years
   (c) 7.7%

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Name________________________________ Alg 2 CC

Date____________________ Unit10 Day9 HW

Solve each equation using natural logarithms. Round your solution to the nearest ten-thousandth.

1. $4e^{2x} - 11 = 17$

$$+11 \quad +11$$
$$\frac{4e^{2x}}{4} = \frac{28}{4}$$
$$e^{2x} = 7$$
$$\frac{2x \ln \cancel{e}}{2} = \frac{\ln 7}{2}$$
$$x = .9730$$

2. $\frac{3e^{5x}}{3} = \frac{9}{3}$

$$e^{5x} = 3$$
$$\frac{5x \ln \cancel{e}}{5} = \frac{\ln 3}{5}$$
$$x = .2197$$

3. How many years, to the nearest tenth of a year, would it take for \$2500 to triple if it is earning a continuous compound interest of 3.5% per year?

$$\frac{7500}{2500} = \frac{2500e^{.035t}}{2500}$$
$$\frac{\ln 3}{.035} = \frac{.035t \ln e}{.035}$$

$t = 31.4$ years

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4. In 2014, the population of Georgia was 9.36 million people. In 2007 it was 8.18 million.

Use $N = N_0e^{kt}$.

a. Determine the value of k, Georgia's relative rate of growth. (to 4 decimal places)

$$\frac{9.36}{8.18} = \frac{8.18e^{7k}}{8.18}$$

$$\frac{\ln\left(\frac{9.36}{8.18}\right)}{7} = \frac{7k\ \overset{1}{\cancel{\ln e}}}{7}$$

$K = .0193$

b. When will Georgia's population reach 10 million people? (Use the value of k from part a) Round your answer to the nearest tenth of a year.

$$\frac{10}{8.18} = \frac{8.18e^{.0193t}}{8.18}$$

$$\frac{\ln\left(\frac{10}{8.18}\right)}{.0193} = \frac{.0193t\ \overset{1}{\cancel{\ln e}}}{.0193}$$

$t = 10.4$ years

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5. Use the formula for continuously compounded interest.

a. If you deposited \$800 in an account paying 4.5% interest compounded continuously, how much money would be in the account in 5 years?

$A = Pe^{rt}$

$A = 800e^{.045(5)}$

$A = \$1001.86$

b. How long, to the nearest year, will it take to double your money?

$2 = e^{.045t}$

$$\frac{\ln 2}{.045} = \frac{.045t\ \overset{1}{\cancel{\ln e}}}{.045}$$

$t = 15.4$ years

15 years

a. If you want to double your money in 9 years, what rate would you need? Round your rate to the nearest tenth of a percent.

$$\frac{1600}{800} = \frac{800e^{9r}}{800} \qquad \frac{\ln 2}{9} = \frac{9r\ \overset{1}{\cancel{\ln e}}}{9}$$

$r = .077$

7.7%

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More Applications of Logarithms

$B(t) = ab^t$ $\quad b = half\left(\frac{1}{2}\right), dbl(2), triple(3)$

$A(t) = a(1 \pm r)^t$ $\quad A = Pe^{rt}$

$A(t) = a\left(1 \pm \frac{r}{n}\right)^{nt}$ $\quad N(t) = N_0 e^{kt}$

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More Applications of Natural Logarithms Unit10 Day10

A hot liquid is cooling in a room whose temperature is constant. Its temperature can be modeled using the exponential function shown below. The temperature, *T*, is in degrees Fahrenheit and is a function of the number of minutes, *m*, it has been cooling.

$$T(m) = 101e^{-0.03m} + 67$$

(a) What was the initial temperature of the water at $m = 0$. Do without using your calculator.

$T(0) = 101\cancel{e^{-.03(0)}}^{1} + 67$

$= 101 + 67 = 168°F$

(b) How do you interpret the statement that $T(60) = 83.7$?

At 60 minutes, the temperature of the liquid is 83.7°F.

(c) Using the natural logarithm, determine algebraically when the temperature of the liquid will reach 100 °F. Show the steps in your solution. Round to the nearest tenth of a minute.

$100 = 101e^{-0.03m} + 67$

$-67 \qquad -67$

$\frac{33}{101} = \frac{101e^{-0.03m}}{101}$

$\ln\left(\frac{33}{101}\right) = \ln e^{-0.03m}$

$\frac{\ln\left(\frac{33}{101}\right)}{-0.03} = \frac{-0.03m \cancel{\ln e}^{1}}{-0.03}$

$m = 37.3$ minutes

(d) On average, how many degrees are lost per minute over the interval $10 \le m \le 30$? Round to the nearest tenth of a degree.

Rate of Change $\frac{\Delta y}{\Delta x}$

$T(30) = 101e^{-0.03(30)} + 67 = 108.06$

$T(10) = 101e^{-0.03(10)} + 67 = 141.82$

$\frac{T(30) - T(10)}{30 - 10} = \frac{108.06 - 141.82}{20}$

$= -1.7$ degrees per minute

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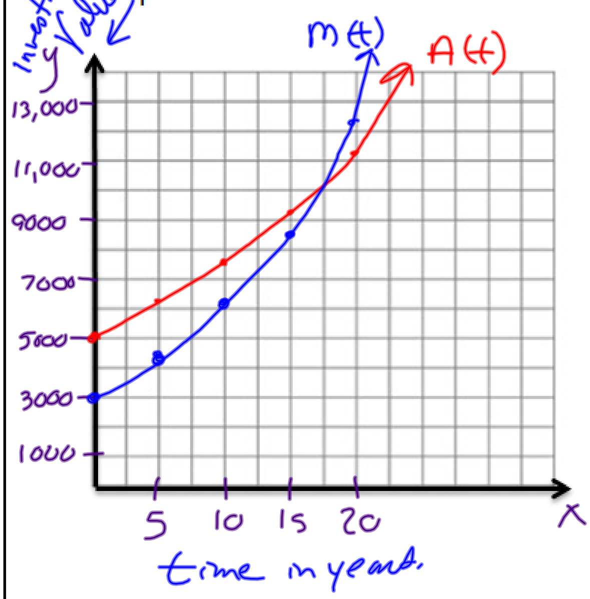
Amy and Mark earned a stipend working at a summer camp. They both decided to invest their earnings. Amy earned \$5000 as the camp director and Mark earned \$3000 as a camp counselor. They both wanted their investment to be compounded continuously so they did their research and invested their money. Amy was able to invest her money at 4% compounded continuously and Mark was able to invest his money at 7% compounded continuously.

1. Write two functions, A(t) and M(t), to represent their investments.
2. Graph each function on the set of axes below.



$A(t) = 5000e^{.04t}$

$M(t) = 3000e^{.07t}$

x	0	5	10	15	20
A(t)	5000	6107	7459	9111	11,128

y	0	5	10	15	20
M(t)	3000	4257	6041	8573	12166

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3. To the nearest year, when will Mark's investment start to exceed Amy's investment.

Using the graph, use $2^{nd} \rightarrow$ Trace $\rightarrow$ Intersect

17 years

4. How long will it take, to the nearest tenth of a year, for the both of them to double their investments? Only algebraic solutions are acceptable.

$A(t) = 5000e^{.04t}$

$10000 = 5000e^{.04t}$

$2 = e^{.04t}$

$\frac{\ln 2}{.04} = \frac{.04t \ln e}{.04}$

17.3 years

$M(t) = 3000e^{.07t}$

$6000 = 3000e^{.07t}$

$2 = e^{.07t}$

$\frac{\ln 2}{.07} = \frac{.07t \ln e}{.07}$

9.9 years

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