Unit 10 Day 10

① (a) $A(t) = 600e^{-.243t}$

(b) See graph on next slide

(c) $t = 7.4$ hours

② (4)

③ (a) $T = 38°C$

(b) 66 minutes

(In the 65 minute)

④ $D = 6.3$ Volts

Jan 30-6:47 PM

Name______________________________ Alg 2 CC

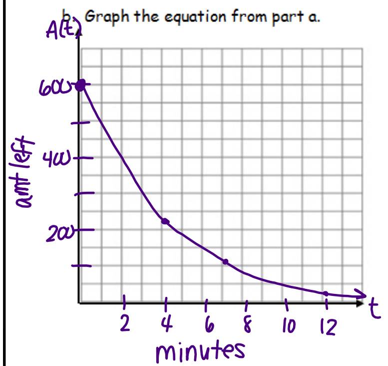
Date____________________ Unit10Day10HW

1. Medications break down in the human body at different rates. The breakdown of a certain medication is represented by the function $A(t) = A_0(e)^{-rt}$, where $A(t)$ is the amount left in the body, A_0 is the initial dosage, r is the decay rate, and t is the time in hours. A patient is given 600 milligrams of a certain medication with a decay rate of 0.243.

a. Write the equation for A(t) that represents the breakdown of the medication.

$A(t) =$ $600e^{-.243t}$

b. Graph the equation from part a.



t	0	4	7	12
A(t)	600	227	109.5	32.49

x-min 0
x-max 10
y-min 0
y-max 700

c. It is safe to take another dose of the medication when you have only 100 milligrams left in your system. Determine, to the *nearest tenth of an hour*, how long a person needs to wait to take another dose of the medication.

$$\frac{100}{600} = \frac{600e^{-.243t}}{600}$$

$$\frac{1}{6} = e^{-.243t}$$

$$\ln\frac{1}{6} = -.243t$$

$$t = \frac{\ln\frac{1}{6}}{-.243}$$

$t \sim 7.4$ hours

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2. Franco invests $4,500 in an account that earns a 3.8% nominal interest rate compounded continuously. If he withdraws the profit from the investment after 5 years, how much has he earned on his investment?

(1) $858.92
(2) $912.59
(3) $922.50
(4) $941.62

$A = 4500e^{.038(5)}$

$A = \$5441.62$

$5441.62 - 4500 = \$941.62$

3. A cup of water at an initial temperature of 76°C is placed in a room at a constant temperature of 20°C. As the water cools, its temperature is described by the equation $T = 20 + 56e^{-0.037t}$, where t is the time elapsed in minutes.

a. What is the temperature of the water one-half hour after the cup was placed in the room, to the nearest degree Celsius?

$t = 30$

$T = 20 + 56e^{-.037(30)}$

$T = 38°$

b. How many minutes will it take for the water to cool off to 25°C?

$25 = 20 + 56e^{-.037t}$

$-20 \quad -20$

$\frac{5}{56} = \frac{56e^{-.037t}}{56}$

$\frac{\ln\left(\frac{5}{56}\right)}{-.037} = \frac{-.037t \cancel{\ln e}}{-.037}$

$t = 66$ minutes

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4. The power output P_0 of an amplifier is given by the formula $P_0 = P_ie^{D/10}$, where P_i is the power input and D is the decibel voltage gain. Determine the decibel voltage gain, to the *nearest tenth*, for an amplifier with a power output of 60W and an input power of 32W.

$\frac{60}{32} = \frac{32e^{D/10}}{32}$

$10 \cdot \ln\left(\frac{60}{32}\right) = \frac{D}{10} \cancel{\ln e} \cdot 10$

$D = 6.3$

The decibel voltage gain is 6.3.

Jan 30-6:50 PM

How many decimal places is to the nearest:

tenth? 1

hundredth? 2

thousandth 3

4 decimal places is to the nearest ten thousandth

Jan 6-7:47 AM

Review for Unit 10 Test

#18

$B(t) = ab^t$

$b = \text{half}\left(\frac{1}{2}\right)^{\frac{t}{\text{every}}}, \text{dbl}(2)^{\frac{t}{\text{every}}}, \text{triple}(3)^{\frac{t}{\text{every}}}$

$A(t) = a(1 \pm r)^t$

$A = Pe^{rt}$

$A(t) = a\left(1 \pm \frac{r}{n}\right)^{nt}$

$N(t) = N_0 e^{kt}$

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Answers to the Review

1. $y = 3^x$
2. d
3. $\{625\}$
4. $(0, \infty)$
   $(-\infty, \infty)$
   $x = 0$
5. $y = \log_2(x-4) - 1$
6. (1) Left 2
   (2) up 3
7. $2 + \frac{1}{3}\log x$
8. $\log x^2\sqrt[4]{y}$
9. 4
10. $3/2$
11. 2
12. 2.681
13. $\ln a^2 b^5$
14. $\ln 2 + \ln x - \ln y - 3\ln z$
15. $\left\{\frac{20}{11}\right\}$
16. $\{.99\}$
17. $\{2.3\}$
18. a. $A = 4200e^{.03t}$
    b. 28.92 years
19. $L = 50(2)^{t/30}$
20. $B(t) = 10{,}000\left(\frac{1}{2}\right)^{t/3}$
21. 52.3 years
22. 33.8 years
23. $k = .0858$

Name________________________________ Alg 2 CC Review for Unit 10 2020

Date_________________

1. What is the inverse of $y = \log_3 x$?

   $3^y = x \longrightarrow y = 3^x$

2. Which statement about the graph of $c(x) = \log_4 x$ *false*?
   a. The domain is the set of positive reals.
   b. The graph contains the point (1,0).
   c. The graph has an asymptote at x=0.
   d. The graph has a y-intercept.

3. If $\log_5 x = 4$, what is the value of x?

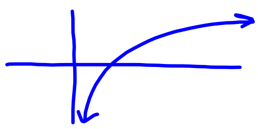
   $5^4 = x \quad x = 625$

4. Given the function $f(x) = \log_2 x$, find

   Domain of f(x) $(0, \infty)$

   Range of f(x) $(-\infty, \infty)$

   Asymptote of f(x) $x = 0$



5. The graph of $y = \log_2 x$ is translated to the right 4 units and down 1 unit. State the equation of the transformed graph.

$y = \log_2(x-4) - 1$

6. Given the following function, $f(x) = \log(x + 2) + 3$, state the transformations that occurred.

1 Left 2
2 Up 3

7. Expand $\log 100\sqrt[3]{x}$

$= \log 100 + \log x^{1/3}$
$= 2 + \frac{1}{3}\log x$

8. Write as a single logarithm

$2\log x + \frac{1}{4}\log y$

$= \log x^2 + \log y^{1/4} = \log x^2\sqrt[4]{y}$

9. Evaluate $\log 10000 = 4$

10. Evaluate $\log_{25} 125$

$\frac{3}{2}$

11. Evaluate $\log_7 49$

2

12. Evaluate and round to the nearest thousandth.

$\log_{15} 1421 \approx 2.681$

13. Rewrite as a single natural logarithm.

$2 \ln a + 5 \ln b$

$\ln a^2 + \ln b^5 = \ln a^2b^5$

14. Expand the natural logarithm.

$\ln \frac{2x}{yz^3}$

$= \ln 2 + \ln x - (\ln y + \ln z^3)$
$= \ln 2 + \ln x - \ln y - 3\ln z$

15. Solve for x using common bases:

$243^{3x} = 81^{x+5}$

$3^{5(3x)} = 3^{4(x+5)}$

$15x = 4x + 20$

$-4x \quad -4x$

$11x = 20$

$x = 20/11$

$\{\frac{20}{11}\}$

16. Solve for x. Round to the nearest hundredth.

$8^{3x} = 475$

$\frac{\log_8 475}{3} = \frac{3x}{3}$

$x = .99$

$\{.99\}$

17. Solve for x using natural logarithms. Round your answer to the nearest tenth.

a. $5e^{2x} + 6 = 26$

$\frac{5e^{2x}}{5} = \frac{20}{5}$

$e^{2x} = 4$

$\frac{\log_e 4}{2} = \frac{2x}{2}$

$x = .7$

$\{.7\}$

b. $3e^{x} - 5 = 25$

$\frac{3e^{x}}{3} = \frac{30}{3}$

$e^{x} = 10$

$\log_e 10 = x$

$x = 2.3$

$\{2.3\}$

18. You put $4200 in a savings account paying 3% interest compounded continuously. $A = Pe^{rt}$

a. Write an equation to model this situation.

b. When will the amount in the savings account reach $10,000? Round your answer to the nearest hundredth of a year.

a. $A = 4200e^{.03t}$

b. $\frac{10{,}000}{4200} = \frac{4200e^{.03t}}{4200}$

$\frac{50}{21} = e^{.03t}$

ln $\frac{\log_e \frac{50}{21}}{.03} = \frac{.03t}{.03}$

$t = 28.92$ years

19. A lady bug population doubles every 30 days. Write an equation, L, in terms of the number of days, t, which would predict the population if there were 50 lady bugs to start.

$L = 50(2)^{t/30}$

20. A certain strain of bacteria has been reduced by half every 3 hours by a new medication being tested by the FDA. Write a function that gives the number of cells that contain the bacteria if there were 10,000 cells to start.

$B(t) = 10{,}000\left(\frac{1}{2}\right)^{t/3}$

21. You put $1200 in a savings account paying 2.1% interest compounded continuously. How long will it take for your savings to triple? Round your answer to the nearest tenth of a year.

$A = Pe^{rt}$

$A = 1200e^{.021t}$

$\frac{3600}{1200} = \frac{1200e^{.021t}}{1200}$

$3 = e^{.021t}$

$\frac{\log_e 3}{.021} = \frac{.021t}{.021}$

$t = 52.3$ years

22. Your investment has been decreasing at a steady rate of 3.2% per year. If you originally invested $3000, using the formula $A = a(1 \pm r)^t$, determine the number of years algebraically that it will take for your investment to reach $1000. Round your answer to the nearest tenth of a year.

$A = 3000(1 - .032)^t$

$\frac{1000}{3000} = \frac{3000(.968)^t}{3000}$

$\frac{1}{3} = (.968)^t$

$\log_{.968}\left(\frac{1}{3}\right) = t$

$t = 33.8$ years

23. In 2005, the deer population in Central New York was estimated to be 102, 541. After a study done in 2015, it was estimated that the deer population grew to 241, 730. Determine the rate of growth using the equation $N = N_0e^{kt}$. Round to the nearest ten-thousandths place.

$\frac{241{,}730}{102{,}541} = \frac{102{,}541e^{k(10)}}{102{,}541}$

$\frac{241{,}730}{102{,}541} = e^{10k}$

$\frac{\log_e\left(\frac{241{,}730}{102{,}541}\right)}{10} = \frac{10k}{10}$

$k = .0858$