Unit 10 Day 6 HW

1. (a) 6191 (b) 2019.9
2. 11.098 hours
3. (a) A = 50(0.98) (b) 34.3 years
4. 5.9 years
5. 10.9 years
6. log 3 + log x + 2log y
7. 2 log a - 1/3 log b
8. $\log \frac{x^2\sqrt{y}}{z^3}$
9. {0.803}

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Name________________________________ Alg 2 C 8

Date____________________ Unit 10 HW Day 6

1. The population of Winnemucca, Nevada, can be modeled by $P(t) = 6191(1.04)^t$ where t is the number of years since 1990.

a. What was the population in 1990?

6191 people

b. In what year will the population be 20,000?

$$\frac{20{,}000}{6191} = \frac{6191(1.04)^t}{6191}$$

$$\log\left(\frac{20{,}000}{6191}\right) = t\log(1.04)$$

$$t = \frac{\log\left(\frac{20000}{6191}\right)}{\log(1.04)}$$

$t = 29.9$ years

$1990 + 29.9 = 2019.9$

2. Growth of a certain strain of bacteria is modeled by the equation $G = A(2.7)^{0.584t}$. In approximately how many hours will 4 bacteria first increase to 2,500 bacteria?

$$\frac{2500}{4} = \frac{4(2.7)^{0.584t}}{4}$$

$$625 = (2.7)^{0.584t}$$

$$\frac{\log 625}{(0.584\log 2.7)} = \frac{0.584t\log(2.7)}{(0.584\log(2.7))}$$

$t = 11.09844215$

$t = 11.098$ hours

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3. A radioactive substance is decaying such that 2% of its mass is lost every year. Originally, there were 50 kilograms of the substance present.

a. Write an equation for the amount, A, of the substance left after t-years.

$$A = 50(1-.02)^t$$
$$A = 50(.98)^t$$

b. Find the amount of time that it takes for only half of the initial amount to remain. Round your answer to the nearest tenth of a year.

$$25 = 50(.98)^t$$
$$\frac{1}{2} = (.98)^t$$
$$\frac{\log(1/2)}{\log(.98)} = \frac{t\log(.98)}{\log(.98)}$$
$$t = 34.3 \text{ years}$$

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4. Emily deposited $5000 in an account at 4% interest. She now has $6300. How many years to the nearest tenth was the money in the account?

$$6300 = 5000(1.04)^t$$
$$\log \frac{63\cancel{00}}{50\cancel{00}} = t\log(1.04)$$
$$\frac{\log \frac{63}{50}}{\log(1.04)} = t \qquad t = 5.9 \text{ years}$$

5. Olivia's parents gave her $10000 to invest for her wedding. Her parents advised her to put it into an account that will pay her 6.4% compounded quarterly. Algebraically determine, to the nearest tenth of a year, how long it would take her to double her initial investment.

$$\frac{20000}{10000} = \frac{10000\left(1+\frac{.064}{4}\right)^{4t}}{}$$
$$2 = \left(1+\frac{.064}{4}\right)^{4t}$$
$$\log 2 = 4t\log\left(1+\frac{.064}{4}\right) \qquad 1+\frac{.064}{4} \rightarrow x$$

or

$$\frac{\log 2}{4\log x} = t \qquad x = 1.016$$
$$t = 10.9 \text{ years}$$

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6. Expand.

a. $\log 3xy^2$

$= \log 3 + \log x + \log y^2$

$= \log 3 + \log x + 2\log y$

b. $\log \frac{a^2}{b^{\frac{1}{3}}}$

$= \log a^2 - \log b^{1/3}$

$= 2\log a - 1/3\log b$

7. Write as a single logarithm.

a. 2log x + 1/2log y -3log z

$= \log x^2 + \log y^{1/2} - \log z^3$

$= \log \frac{x^2\sqrt{y}}{z^3}$

8. Solve. Round the solution to the nearest thousandth.

$6^{3n} + 4 = 79$

$-4 \quad -4$

$6^{3n} = 75$

$\log_6 75 = 3n$

$n = \frac{\log_6 75}{3} = 0.803$

QUIZ

More Solving Word Problems with Logarithms

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More Solving of Word Problems with Logarithms Unit 10 Day 7

"Half Lives", "Doubling Time", "Tripling Time"

We have learned about the base of a growth or decay exponential equation. Fill in the boxes below to show what the base needs to be to demonstrate growth or decay.

Now let us look at each of the following models in terms of the given base.

There is a special base that we have to consider. What if we are asked to evaluate something that has a half-life every so many days? Now we need to have a special base of $\frac{1}{2}$. The words that follow the word **EVERY** will be what we use for the exponent. The "every whatever" will always be the denominator of the exponent. Let us fill in the box below and represent this general situation.

The same holds true for a quantity being doubled, tripled, etc. Let us fill in the box below and represent this general situation.

$B(t) = a\,b^t$

Growth	Decay	Special "half-life"	Double/Triple/ etc.
$(b)^t$ $b > 1$	$(b)^t$ $0 < b < 1$	$\left(\frac{1}{2}\right)^{t/\text{every}}$	$b = 2 \rightarrow 2^{t/\text{every}}$ $b = 3 \rightarrow 3^{t/\text{every}}$

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Let us represent the following situations according to what we just discovered above.

Double every 3 days $(2)^{t/3}$ $t = \#$ days

Triple every 15 minutes $(3)^{t/15}$ $t = \#$ mins.

Half every 3.7 years $\left(\frac{1}{2}\right)^{t/3.7}$ $t = \#$ years

Remember that whatever follows the every is always the denominator of the exponent.

1. If a population of honeybees doubles every 5 years how many years to the *nearest* tenth *of a year* will it take the population to increase by 10 times the original amount?

$2^{t/5}$ $t = $ yrs $t = ?$

Isolate

$B(t) = a(2)^{t/5}$

$\frac{10a}{a} = \frac{a(2)^{t/5}}{a}$

$10 = (2)^{t/5}$

$5\left(\frac{t}{5}\right) = (\log_2 10)5$

$t = 5\log_2 10$

$t = 16.609$

≈ 16.6 years

$B(t) = 1(2)^{t/5}$

$\frac{10}{1} = \frac{1(2)^{t/5}}{1}$

$10 = (2)^{t/5}$

$\log 10 = \log$ ___

$\log 10 = \frac{t}{5}\log 2$

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2. One of the medical uses of Iodine-131 (I-131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I-131 is every 8.02 days. A patient is injected with 20 milligrams of I-131. Determine, to the nearest hundredth of a day, the amount of time needed before the amount of I-131 in the patient's body is approximately 7 milligrams.

$t/8.02$

$B(t) = 20(.5)^{t/8.02}$

$\frac{7}{20} = \frac{20(.5)^{t/8.02}}{20}$

$.35 = (.5)^{t/8.02}$

$8.02\left(\frac{t}{8.02}\right) = (\log_{.5}(.35))8.02$

$t = 8.02\log_{.5}(.35)$

$t = 12.15$ days

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3. A colony of bacteria grows according to the law of uninhibited growth. The number of bacteria triple every 3 hours. Write a function that gives the number of cells in the culture if there are 10,000 cells to start. How long, to the nearest tenth of an hour, will it take the culture to reach 156,000 cells?

$$B(t) = 10{,}000(3)^{t/3}$$

$$\frac{156{,}000}{10{,}000} = \frac{10{,}000(3)^{t/3}}{10{,}000}$$

$$15.6 = 3^{t/3}$$

$$3(\log 15.6) = \left(\frac{t}{3}\log 3\right)3$$

$$3\log 15.6 = t\log 3$$

$$t = \frac{3\log 15.6}{\log 3} \approx 7.5 \text{ hours}$$

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As we looked at in the last unit, sometimes we have to change the base of an equation using the power rule. Let us practice some word problems with this concept.

$(b)^{at} = (b^a)^t = (b^t)^a$

4. The population of killer bees in a colony is increasing according to the following formula: $B(t) = 62{,}456(0.76)^{-3t}$ where $B(t)$ is the population of bees in the colony t years after 2017. The equation can be written in the form $B(t) = 62{,}456(x)^t$. What is the value of x to the nearest thousandth? 3 dec

$B(t) = 62{,}456(.76)^{-3t}$

$B(t) = 62{,}456(.76^{-3})^t$

$(.76^{-3}) = 2.278$

New EQ: $B(t) = 62{,}456(2.278)^t$

Using the new equation, what is the population in 2020? Round your answer to the nearest hundredths place. $t = 2020 - 2017 = 3$

$B(3) = 62{,}456(2.278)^3 = 738304.18$

Using the new equation, when will the population, rounded to the nearest hundredth, reach a half of a million killer bees? 500,000

$$500{,}000 = 62456(2.278)^t$$

$$\frac{500{,}000}{62456} = (2.278)^t$$

$$t = \log_{2.278}\left(\frac{500{,}000}{62456}\right) \approx 2.53 \text{ years}$$

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