Unit 10 Day 7 HW

Test next Wednesday

1. $f(n) = 6(2)^{4n}$
2. (3)
3. ~~5200 years~~ 5169.53 years
4. (a) 16600 people (b) 11.3 years
5. t = 23.22 minutes
6. h = 10.3002 t = 18.6 hours

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Name________________________________ Alg 2 CC

Date________________ Unit 10 Day 7 HW

1. A computer application generates a sequence of musical notes using the function $f(n) = 6(16)^n$ where n is the number of the note in the sequence and $f(n)$ is the note frequency in hertz. Write a function that will generate the same note sequence as $f(n)$ with a base of 2.

$f(n) = 6(2^4)^n$ $16 = 2^4$

$f(n) = 6(2)^{4n}$

2. Iridium-192 is an isotope of iridium and has a half-life of 73.83 days. If a laboratory experiment begins with 100 grams of Iridium-192, the number of grams, A, of Iridium-192 present after t days would be

$A = 100\left(\frac{1}{2}\right)^{\frac{t}{73.83}} = 100\left(\frac{1}{2}\right)^{\frac{1}{73.83}t} = 100(.990656)^t$

Which equation approximates the amount of Iridium-192 present after t days?

$\left(\frac{1}{2}\right)^{\frac{1}{73.83}}$

(1) $A = 100\left(\frac{73.83}{2}\right)^t$ (3) $A = 100(0.990656)^t$

(2) $A = 100\left(\frac{1}{147.66}\right)^t$ (4) $A = 100(0.116381)^t$

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3. The half-life of radium is every 1690 years. If 10 grams are present now, how long will it take to the nearest hundredth of a year, to have 1.2 grams of radium?

5169.53

$$\frac{1.2}{10} = \frac{10\left(\frac{1}{2}\right)^{t/1690}}{10}$$

$$.12 = \left(\frac{1}{2}\right)^{t/1690}$$

$$\frac{\log(.12)}{\log(\frac{1}{2})} = \frac{\frac{t}{1690}\log\left(\frac{1}{2}\right)}{\log(\frac{1}{2})}$$

$$\frac{\log(.12)}{\log(\frac{1}{2})} \cdot 1690 = \frac{t}{1690} \cdot 1690$$

4. The current population of Little Pond, New York is 20,000. The population is decreasing, as represented by the formula $P = A(1.3)^{-0.234t}$, where P = final population, t = time in years, and A = initial population.

a. What will the population be 3 years from now? Round your answer to the nearest hundred people.

$$P = 20{,}000(1.3)^{-.234(3)}$$

$$= 16635.7$$

$$= 16600 \text{ people}$$

b. To the nearest tenth of a year, how many years will it take the population to reach half the present population?

$$\frac{1}{2} = (1.3)^{-.234t}$$

$$\frac{\log(\frac{1}{2})}{-.234\log(1.3)} = \frac{-.234t\log(1.3)}{-.234\log(1.3)}$$

$$t = 11.29 \approx 11.3 \text{ years}$$

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5. The number of bacteria in a petri dish doubles every 10 minutes. How many minutes to the nearest hundredth of a minute will it take for the bacteria to be 5 times the original amount?

$$5 = 1(2)^{t/10}$$

$t = 23.22$ minutes

$$\frac{\log 5}{\log 2} = \frac{\frac{t}{10}\log 2}{\log 2}$$

$$10 \cdot \frac{\log 5}{\log 2} = \frac{t}{10} \cdot 10$$

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6. A radioactive substance has a mass of 140 g at 3 p.m. and 100 g at 8 p.m.

Write an equation in the form $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$ that models this situation, where h is the constant representing the number of hours in the half-life, A_0 is the initial mass, and A is the mass t hours after 3 p.m.

Using this equation, solve for h, to the nearest ten thousandth.

8 PM
−3 PM
$t = 5$

$$\frac{100}{140} = \frac{140\left(\frac{1}{2}\right)^{5/h}}{140}$$

$$\frac{100}{140} = \left(\frac{1}{2}\right)^{5/h} \qquad \log_{\frac{1}{2}}\left(\frac{100}{140}\right) = \frac{5}{h}$$

$$\frac{\log\left(\frac{10}{14}\right)}{\log\left(\frac{1}{2}\right)} = \frac{\frac{5}{h}\cancel{\log\left(\frac{1}{2}\right)}}{\cancel{\log\left(\frac{1}{2}\right)}}$$

Determine when the mass of the radioactive substance will be 40 g. Round your answer to the nearest tenth of an hour.

$$\frac{.4854268272}{1} = \frac{5}{h}$$

$$\frac{.4854268272h}{.4854268272} = \frac{5}{.4854268272}$$

$$h = 10.3002$$

$$\frac{40}{140} = \frac{140\left(\frac{1}{2}\right)^{t/10.3002}}{140}$$

$$\frac{\log\left(\frac{4}{14}\right)}{\log\left(\frac{1}{2}\right)} = \frac{\frac{t}{10.3002}\log\left(\frac{1}{2}\right)}{\log\left(\frac{1}{2}\right)}$$

$$\frac{\log\left(\frac{4}{14}\right)}{\log\left(\frac{1}{2}\right)} \cdot 10.3002 = \frac{t}{10.3002} \cdot 10.3002$$

$$t = 18.6 \text{ hours}$$

$$\boxed{\frac{4}{14} = \left(\frac{1}{2}\right)^{t/10.3002}}$$

$$\frac{t}{10.3002} \times \frac{\log_{\frac{1}{2}}\left(\frac{4}{14}\right)}{1}$$

$$t = 10.3002 \log_{\frac{1}{2}}\left(\frac{4}{14}\right)$$

$$t = 18.6 \text{ hrs.}$$

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https://www.youtube.com/watch?v=R0oUeLQIbIk (2 mins)

The Number e

The number e. $3! = 3 \cdot 2 \cdot 1 = 6$

e is a special irrational number. e stands for Euler' number. Leonhard Euler (oy-ler) was a Swiss mathematician & physicist.

$$e = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \dots$$

$e \approx 2.72\dots$ (2nd ÷ or 2nd ln)

Evaluate the following to the nearest hundredth:

1. $e^{2.47} = 11.82$

2. $e^{3.51} = 33.45$

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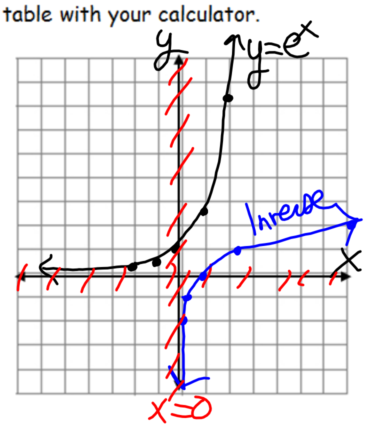
Graph $y = e^x$ and its inverse. For graph of e, create a table with your calculator.

$x = e^y$

x	y
-2	.14
-1	.37
0	1
1	2.72
2	7.39

x	y
.14	-2
.37	-1
1	0
2.72	1
7.39	2

$y=0$



$\{x \mid x \in \mathbb{R}\}$

Domain of $y = e^x$ $(-\infty, \infty)$

$\{y \mid y > 0\}$

Range of $y = e^x$ $(0, \infty)$

Asymptote of $y = e^x$ $y=0$

Domain of Inverse $(0, \infty)$

Range of Inverse $(-\infty, \infty)$

Asymptote of Inverse $x=0$

$y = e^x$

Inverse EQ.

$x = e^y$

$y = \log_e x$

$y = \ln x$

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Because of the importance of $y = e^x$, its **inverse**, known as the **natural logarithm** is also

THE NATURAL LOGARITHM

The inverse of $y = e^x$: $y = \ln x$ $(y = \log_e x)$

important. The natural logarithm, like all logarithms, gives an exponent as its output. In fact, it gives the power that we must raise **e** to in order to get the input.

$\log_3 3 = 1$

$y = e^x$

Equation of the inverse: → $y = \ln x$

log base e → ln $(= \log_e)$

$e^{\square} = e$

ln e = $\log_e e$ = 1

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Write each expression as a single natural logarithm.

1. $3 \ln 5 + 1/2 \ln x$
$= \ln 5^3 + \ln x^{1/2}$
$= \ln 125\sqrt{x}$

2. $\ln 24 - \ln 6$
$= \ln \frac{24}{6} = \ln 4$

$3 + 5 - 4 = 4$
$3 - 4 + 5 = 4$
$-1 \quad +5$

3. $1/3(\ln x + \ln y) - 4 \ln z$
$\ln\sqrt[3]{x} + \ln\sqrt[3]{y} - \ln z^4$
$\ln\left(\frac{\sqrt[3]{xy}}{z^4}\right)$

4. $\ln a - 2 \ln b + \frac{1}{2} \ln c$
$= \ln a + \ln\sqrt{c} - \ln b^2$
$= \ln \frac{a\sqrt{c}}{b^2}$

$\frac{a}{b^2} \cdot \frac{\sqrt{c}}{1}$

Expand each natural logarithm.

5. $\ln \frac{a^2b^3}{\sqrt{c}} = \ln a^2 + \ln b^3 - \ln c^{1/2}$
$= 2\ln a + 3\ln b - \frac{1}{2}\ln c$

6. $\ln (2x)^2 = 2 \ln 2x = 2\ln 2 + 2\ln x$
$\ln 4x^2 = \ln 4 + 2\ln x$

7. $\ln 49xyz$
$= \ln 49 + \ln x + \ln y + \ln z$

8. $\ln \frac{\sqrt[3]{r}}{st} = \ln r^{1/3} - (\ln s + \ln t)$
$= \frac{1}{3}\ln r - \ln s - \ln t$

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Solve. Round to the nearest hundredth where necessary. Depending on what method you chose earlier in the unit, you will either put a natural logarithm on both sides to solve for the variable or change the equation from exponential form to logarithmic form and use the log base feature on your graphing calculator to solve for the variable. Before using either method, you must isolate the base e.

Isolate

9. $e^x = 18$

$x = \ln 18$

$x \approx 2.89$

10. $e^{x/5} + 4 = 7$

$-4 \quad -4$

$e^{x/5} = 3$

$5\left(\frac{x}{5}\right) = (\ln 3)5$

$x = 5\ln 3$

$x \approx 5.49$

11. $\frac{10}{5} = \frac{5e^k}{5}$

$2 = e^k$

$k = \ln 2$

$k = .69$

12. $7 - 2e^{x/2} = 1$

$-7 \quad\quad -7$

$\frac{-2e^{x/2}}{-2} = \frac{-6}{-2}$

$e^{x/2} = 3$

$\frac{x}{2} = \ln 3$

$x = 2\ln 3$

$x \approx 2.20$

$x \approx 2.20$

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