Unit 10 Day 8 HW

Do Warm-up in Notes
Wednesday Test

1. $x = .5$
2. $x = 2.71$
3. $x = 0.92$
4. $x = 1.91$
5. $x = 17.33$

6. $\ln 243$
7. $\ln \frac{x^4}{\sqrt[3]{y}}$
8. $\ln \frac{x^2\sqrt{y}}{zw^3}$

11. $3\ln x + \frac{1}{2}\ln y$
12. $2\ln a + 3\ln b - \ln c - \frac{1}{2}\ln d$
13. $3\ln x - 2$
14.
a) $B = 100(2)^{t/30}$
b) 108.62 hours

Jan 23-7:00 PM

Name______________________________ Alg 2 CC

Date______________________ Unit 10 HW Day 8

1. Using logarithms, find the value of x to the nearest tenth. $x = 0.5$

$7^{(x+1)} = 18.6$

$\frac{(x+1)\log 7}{\log 7} = \frac{\log 18.6}{\log 7}$

$x + 1 = 1.5022$
$-1 \quad -1$
$x = .5$

Use natural logarithms to solve each equation. Round your answer to the nearest hundredth.

2. $e^x = 15$

$x \cancel{\ln e} = \ln 15$
$x = 2.71$

3. $\frac{4e^x}{4} = \frac{10}{4}$

$e^x = \frac{5}{2}$
$x \cancel{\ln e} = \ln(5/2)$
$x = .92$

Jan 23-7:00 PM

4. $e^{x+2} = 50$

$(x+2)\cancel{\ln e}^{\,1} = \ln 50$

$-2 \qquad -2$

$x = 1.91$

~~$\ln x = -2$~~

~~$\frac{}{4} \quad \frac{}{4}$~~

~~$\log_e x = -1/2$~~

~~$e^{-1/2} = x$~~

~~$x = .61$~~

5. $e^{\frac{x}{5}} - 4 = 28$

$+4 \quad +4$

$e^{x/5} = 32$

$\frac{x}{5}\cancel{\ln e}^{\,1} = \ln 32$

$x = 5 \cdot \ln 32$

$x = 17.33$

$\frac{x}{5} = \ln 32$

$x = 5\ln 32$

≈ 17.33

~~7. $\ln 2x = 6$~~

~~$\log_e 2x = 6$~~

~~$e^6 = 2x$~~

~~$\frac{}{2} \quad \frac{}{2}$~~

~~$x = 201.71$~~

Write each expression as a single natural logarithm.

6. $3\ln 3 + \ln 9$

$\ln 3^3 \cdot 9$

$\ln 243$

7. $4\ln x - 1/3\ln y$

$\ln \frac{x^4}{y^{1/3}} = \ln \frac{x^4}{\sqrt[3]{y}}$

8. $2\ln x + \frac{1}{2}\ln y - \ln z - 3\ln w$

$\ln x^2 + \ln y^{1/2} - (\ln z + \ln w^3)$

$= \ln \frac{x^2\sqrt{y}}{zw^3}$

Jan 23-7:02 PM

Expand each natural logarithm.

11. $\ln x^3\sqrt{y}$

$= 3\ln x + \ln y^{1/2}$

$= 3\ln x + \frac{1}{2}\ln y$

12. $\ln \frac{a^2b^3}{c\sqrt{d}}$

$= \ln a^2 + \ln b^3 - (\ln c + \ln d^{1/2})$

$= 2\ln a + 3\ln b - \ln c - \frac{1}{2}\ln d$

13. $\ln \frac{x^3}{e^2} = \ln x^3 - \ln e^2$

$= 3\ln x - 2\cancel{\ln e}^{\,1}$

$= 3\ln x - 2$

Jan 23-7:00 PM

14. Biologists are studying a new strain of bacteria. They create a culture in a petri dish with 100 of the bacteria and anticipate that the number of bacteria will double every 30 hours.

a. Write an equation for the number of bacteria, B, in terms of the number of hours, t, since the experiment began.

b. When will the bacteria, to the ***nearest hundredth*** of an hour, reach 1230 in the petri dish?

a. $B = 100(2)^{t/30}$

b. $\frac{1230}{100} = \frac{100(2)^{t/30}}{100}$

$12.3 = (2)^{t/30} \longrightarrow \log 12.3 = \frac{t}{30} \log 2$

$\log_2 12.3 = \frac{t}{30}$

$t = 30 \cdot \log_2 12.3$

$t = 108.62$ hours

Applications of Natural Logarithms

Applications of Natural Logarithms Unit10 Day9

$\ln e = \log_e e = 1$

Warm-up:
Solve and round to the nearest hundredth

$$\frac{1600}{4} = \frac{4e^{.045t}}{4}$$

$$400 = e^{.045t}$$

alt. 1

$.045t = \ln 400$

$t = \frac{\ln 400}{.045}$

alt 2

$\ln 400 = \ln e^{.045t}$

$\ln 400 = .045t \cancel{\ln e}$

$t = \frac{\ln 400}{.045}$

$t \approx 133.14$

Money compounded continuously: $\boxed{A = Pe^{rt}}$

$.045t = \log_e 400$

$\frac{}{.045}$

Where A = ending amount

P = principle or initial amount

r = rate as a decimal

t = time, usually years.

Exponential Growth or Decay: $N(t) = N_0 e^{kt}$

Where $N(t)$ = Ending amount (Not $)

N_0 = initial amount

k = growth or decay rate

t = time, usually years.

Examples:

1. A certain city has a population of $\boxed{P(t) = 142{,}000e^{0.014t}}$ where t is the time in years and $t = 0$ is the year 2010

a. What is the population in 2010? → 142,000 $P(0) = 142{,}000$

b. What is the population in 2020?

c. In what year will the city have a population of 200,000? t?

(b) $t = 10$

$P(10) = 142{,}000e^{.014(10)} = 163{,}338$

(c) $\frac{200{,}000}{142{,}000} = \frac{142{,}000e^{.014t}}{142{,}000}$

$\frac{100}{71} = e^{.014t}$

$\frac{.014t}{.014} = \frac{\ln\left(\frac{100}{71}\right)}{.014}$

$t = 24.46 + 2010 = 2034.46$

In year 2034

Exp. function : $B(t) = ab^t$

$A(t) = a(1 \pm r)^t$

* $A(t) = a(1 \pm \frac{r}{n})^{nt}$

base = half (1/2), dbl (2), triple (3)

* $A = Pe^{rt}$

$N(t) = N_0 e^{kt}$ > Compounding Continuously

2. Sam invested a ~~sum of~~ money in a certificate of deposit that pays 8% interest compounded continuously. If he made the investment on January 1, 1997 and the account was worth $10,000 on January 1, 2016, what was the original amount in the account?

$\rightarrow r = .08$

$A = Pe^{rt}$ $t = 19 = 2016 - 1997$

$10{,}000 = Pe^{.08(19)}$

$P = \frac{10{,}000}{e^{.08(19)}} = \$2187.12$

3. Mike deposited ~~some~~ money in a bank account that earns 5.6% interest compounded continuously. How long will it take to double the money in his account?

$A = 2$

$P = 1$

$r = .056$

$t = ?$

$A = Pe^{rt}$

$2 = 1 \cdot e^{.056t}$

$\ln 2 = .056t \ln e$

$t = \frac{\ln 2}{.056}$ = 12.38 years

In the 12th year.

12 years & 138 days

4. DDT is an insecticide that has been used by farmers. It decays slowly and is sometimes absorbed by plants that animals and humans eat. DDT absorbed in the mud at the bottom of a lake is degraded into harmless products by bacterial action. Experimental data shows that 10% of the initial amount is eliminated in 5 years. If k = -0.0211

$N(t)=N_0e^{kt}$

a. How much of the original amount of DDT is left after 10 years?

$N(10) = 1e^{-.0211(10)}$

$N(10) = .81$ $\therefore$ 81% is left.

b. The US Environmental Protection Agency banned almost all use of DDT in the US in 1972. If none has been used near the lake since then, in what year will the concentration of DDT fall below 25%?

no one

t

$.25 = 1e^{-.0211t}$

$\ln .25 = -.0211t \ln e$

$t = \frac{\ln .25}{-.0211} = 65.701$

1972

In 2037

Jan 23-5:57 PM

Jan 23-5:58 PM