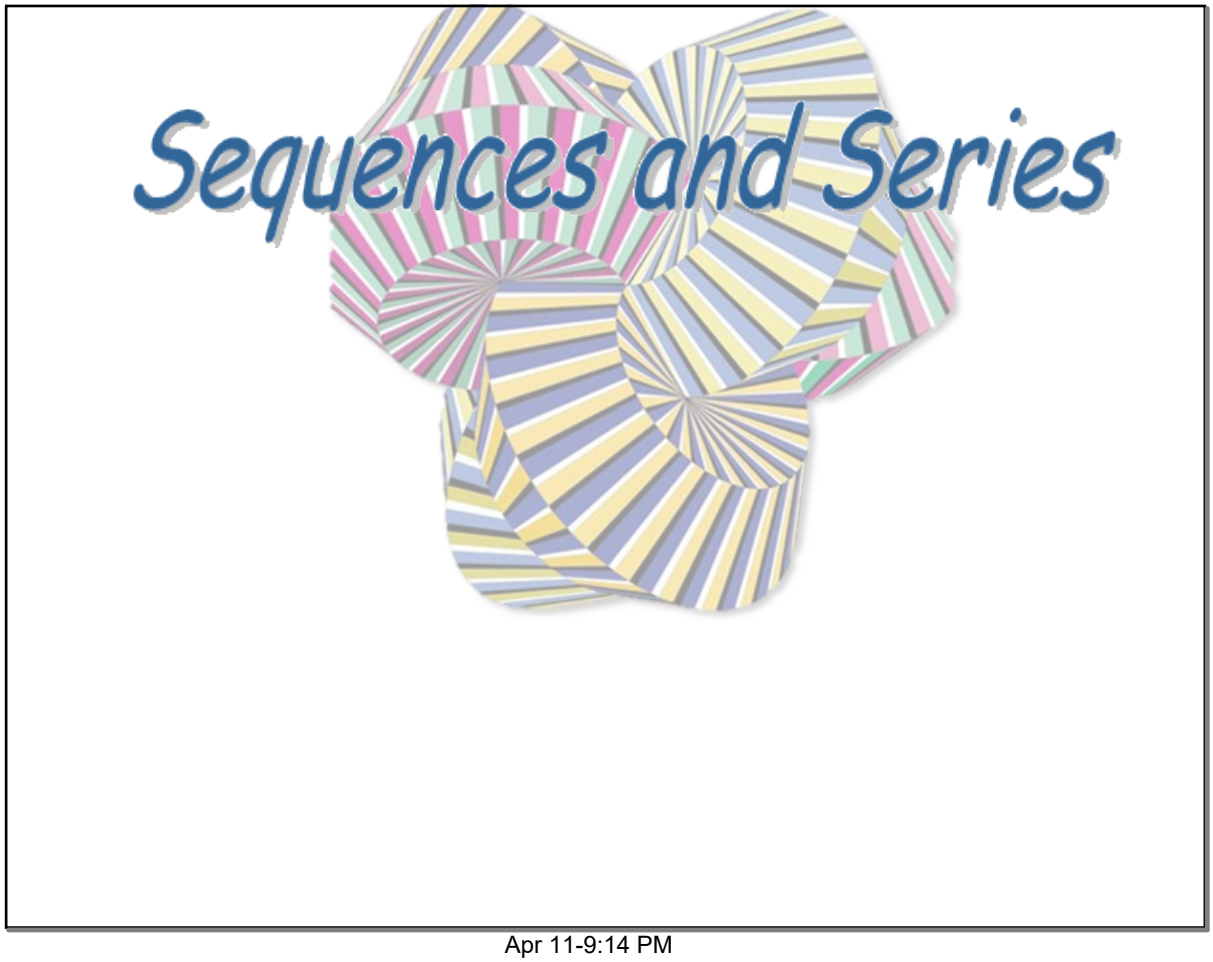

Apr 11-9:14 PM

Introduction to Sequences & Arithmetic Sequences **Day 1**

A Sequence is an ordered set of numbers or other items. Each number in the sequence is called a term. For example, in the sequence 4, 7, 10, 13, 16, ..., the second term is 7. A sequence can be infinite (without end) or finite (limited number of terms). Each term in the sequence can be paired with a position number, and these pairings establish a function whose domain is the set of positive position numbers and whose range is the set of terms, as shown below. The position numbers are consecutive integers that typically start at either 0 or 1.

Position number	n	1	2	3	4	5
Term of sequence	f(n)	4	7	10	13	16

(*) For this sequence, we write f(4) = 13, which can be interpreted as "the fourth term of the sequence is 13."

4, 7, 10, 13, 16 ... Can you find a pattern? Add 3

In this sequence, each term is 3 more than the previous term.

This is an Arithmetic sequence because successive terms differ by the **same** number, called the common difference (d and d ≠ 0).

$a_1, a_2, a_3,$

3 is called the common difference (d)

4 is called the first term (a_1)

7 is called the second term (a_2)

16 is called the fifth term (a_5)

Apr 11-9:17 PM

Now, we'll find a formula (rule) for the n^{th} term (a_n).

1st term:	a_1 = 4	= 4
n=2 2nd term:	a_2 = 4+3(1)	= 7
n=3 3rd term:	a_3 = 4+3(2)	= 10
4th term:	a_4 = 4+3(3)	= 13
5th term:	a_5 = 4+3(4)	= 16
nth term:	a_n = 4+3(n-1)	

$a_n = 4 + 3n - 3$

$a_n = 3n + 1$

100th term: $n = 100$, $a_{100} = 3(100) + 1 = 301$

Arithmetic Rule:

$a_n = a_1 + d(n-1)$

In Function Notation:

$f(n) = a_1 + d(n-1)$

1. Is the sequence arithmetic? If so, find d, an n^{th} term formula (a_n) and the next three terms.

a. 1.9, 1.2, 0.5, -0.2, -0.9... as

$d = a_2 - a_1 = 1.2 - 1.9 = -.7$

Check d w/ rest.

Yes

$a_n = 1.9 - .7(n-1)$

$a_n = 1.9 - .7n + .7$

$a_n = -.7n + 2.6$

$a_6 = 1.9 - .7(5) = -1.6$

$a_7 = 1.9 - .7(6) = -2.3$

$a_8 = 1.9 - .7(7) = -3.0$

b. $\frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32}, \frac{3}{64}...$ ÷2 × 1/2

× 1/2

No, not arithmetic. There is no common difference. The pattern is mult. by 1/2 But it is a sequence

Apr 11-9:17 PM

The previous examples used an explicit formula $a_n = a_1 + d(n-1)$ which defines the nth term of a sequence as a function of n. Sequences can also be described by using a recursive formula which defines the nth term of a sequence as a function of one or more previous terms. On the previous page, we wrote the explicit formula f(n) = 4 + 3(n-1) for the arithmetic sequence 4, 7, 10, 13, 16, 19, 22, 25, 100th term

Use the following recursive formula to find the first 4 terms of the same arithmetic sequence.

f(n) = f(n-1) + 3 with f(1) = 4

f(1) = 4

f(2) = f(2-1)+3 = f(1)+3 = 4+3 = 7

f(3) = f(2)+3 = 7+3 = 10

f(4) = f(3)+3 = 10+3 = 13

Apr 11-9:18 PM

2. Find an explicit and recursive formula rule for the n^{th} term (a_n). Then find the next term with your recursive formula.

a. 9.2, 9.15, 9.1, 9.05, 9...

$d = a_2 - a_1 = 9.15 - 9.2$

$d = -.05$

Explicit

$a_n = a_1 + d(n-1)$

$a_n = 9.2 - .05(n-1)$

$a_n = 9.2 - .05n + .05$

$a_n = 9.25 - .05n$

$a_n = -.05n + 9.25$

Recursive

$a_n = a_{n-1} - .05, a_1 = 9.2$

$a_6 = a_5 - .05 = 9 - .05 = 8.95$

b. $\frac{4}{3}, \frac{5}{3}, 2, \frac{7}{3}, \frac{8}{3}, \ldots$

$d = a_2 - a_1 = \frac{5}{3} - \frac{4}{3} = \frac{1}{3}$

$a_n = a_1 + d(n-1)$

$a_n = \frac{4}{3} + \frac{1}{3}(n-1)$ — explicit

$a_n = \frac{4}{3} + \frac{1}{3}n - \frac{1}{3}$

$a_n = \frac{1}{3}n + 1$

Recursive.

$a_n = a_{n-1} + \frac{1}{3}, a_1 = \frac{4}{3}$

$a_6 = a_5 + \frac{1}{3} = \frac{8}{3} + \frac{1}{3} = \frac{9}{3} = 3$

Apr 11-9:18 PM

3. Find the 12^{th} term of the arithmetic sequence by using a formula.

a. 32, 25, 18, 11, 4...

$d = a_2 - a_1 = 25 - 32 = -7$

$a_n = 32 - 7(n-1)$

$a_n = 32 - 7n + 7$

$a_n = -7n + 39$

$a_{12} = -7(12) + 39$

$a_{12} = -45$

b. -9, -7, -5, -3, -1...

$d = -7 - -9 = 2$

$a_n = -9 + 2(n-1)$

$a_n = -9 + 2n - 2$

$a_n = 2n - 11$

$a_{12} = 2(12) - 11$

$a_{12} = 13$

Apr 11-9:18 PM

4. Find the 6th term of the arithmetic sequence using the given terms and a formula. Do b first.

a. $a_8 = -4$ and $a_{11} = 14$

$d = \frac{a_{11} - a_8}{11-8} = \frac{14 - -4}{3} = \frac{18}{3} = 6$

6th

$a_n = a_1 + d(n-1)$

$a_8 = a_6 + d(n-1)$

$-4 = a_6 + 6(2)$

$-4 = a_6 + 12$

$-12 \quad -12$

$a_6 = -16$

b. $a_3 = 20.5$ and $a_8 = 13$

$d = \frac{a_8 - a_3}{8-3} = \frac{13 - 20.5}{5} = \frac{-7.5}{5} = -1.5$

6th

$a_n = a_1 + d(n-1)$

$a_6 = a_3 + d(6-3)$

$a_6 = 20.5 - 1.5(3)$

$a_6 = 16$

Apr 11-9:19 PM

Apr 11-9:21 PM