

3. $2x(x - 2)$
4. $4mn(m - 2n + 3mn)$
5. $(x + 1)(x - 1)$
6. $(5x - 6y)(5x + 6y)$
7. $(x - 3)(x + 4)(x - 4)$
8. $5x^3(x^3 + 3)$
9. $(11x + 5y)(11x - 5y)$
10. $(16x^2 + 1)(4x + 1)(4x - 1)$
11. $(x + 4)(x + 3)(x - 3)$
12. $6rs(5s - 4r + 1)$
13. $y^a(y^a + 1)(y^a - 1)$
14. $9(3x + y)(3x - y)$
15. $-11x^3 + x^2 + 8x + 4$
16. $-3x^3y^4 - 6x^4y^2 + 12x^2y^3 - 9x^2y^2$
17. $10a^2 - 33ab + 27b^2$
18. $-7x^3 + 26x^2 - 10x - 15$

1 & 2. See next page.

1-4 HW Answer Key

1. Algebraically prove why $a^2 - b^2 = (a+b)(a-b)$ by simplifying the right side.

$$\begin{aligned} a^2 - b^2 &= a^2 + ab - ab - b^2 \\ a^2 - b^2 &= a^2 - b^2 \end{aligned}$$

2. Using $a=5$ and $b=2$, show that $a^2 - b^2 = (a+b)(a-b)$ is true.

$$\begin{aligned} (5)^2 - (2)^2 &= (5+2)(5-2) \\ 25 - 4 &= (7)(3) \\ 21 &= 21 \end{aligned}$$

Factor completely:

3. $2x^2 - 4x$ GCF
 $= 2x(x-2)$

4. $4m^2n - 8mn^2 + 12m^2n^2$ GCF
 $= 4mn(m-2n+3mn)$

5. $x^2 - 1$ DOTS
 $= (x+1)(x-1)$

6. $25x^2 - 36y^2$ DOTS
 $= (5x-6y)(5x+6y)$

7. $x^3 - 3x^2 - 16x + 48$ Grouping
 $= x^2(x-3) - 16(x-3)$
 $= (x-3)(x^2-16)$
 $= (x-3)(x+4)(x-4)$

8. $5x^6 + 15x^3$ GCF
 $= 5x^3(x^3+3)$

9. $121x^2 - 25y^2$ DOTS
 $= (11x+5y)(11x-5y)$

10. $256x^4 - 1$ DOTS
 $= (16x^2+1)(16x^2-1)$
 $= (16x^2+1)(4x+1)(4x-1)$

11. $x^3 + 4x^2 - 9x - 36$ Group DOTS
 $= (x+4)(x^2-9)$
 $= (x+4)(x+3)(x-3)$

12. $30rs^2 - 24r^2s + 6rs$ GCF
 $= 6rs(5s-4r+1)$

13. $y^{3a} - y^a$ GCF/DOTS
 $= y^a(y^{2a}-1)$
 $= y^a(y^{a+1})(y^a-1)$

14. $81x^2 - 9y^2$ GCF/DOTS
 $= 9(9x^2 - y^2)$
 $= 9(3x+y)(3x-y)$

Simplify by rewriting after
 Rewrite each polynomial in standard form ~~by~~ applying the operations in the appropriate order.

$$\begin{aligned} 15. (2x^2 + 3x + 4) - (11x^3 + x^2 - 5x) \\ = 2x^2 + 3x + 4 - 11x^3 - x^2 + 5x \\ = -11x^3 + x^2 + 8x + 4 \end{aligned}$$

$$\begin{aligned} 16. -3xy^2(2x^3 + x^2y^2 - 4xy + 3x) \\ = -6x^4y^2 - 3x^3y^4 + 12x^2y^3 - 9x^2y^2 \\ = -3x^3y^4 - 6x^4y^2 + 12x^2y^3 - 9x^2y^2 \end{aligned}$$

$$\begin{aligned} 17. (2a - 3b)(5a - 9b) \\ = 10a^2 - 18ab - 15ab + 27b^2 \\ = 10a^2 - 33ab + 27b^2 \end{aligned}$$

$$\begin{aligned} 18. (5 + 5x - 7x^2)(x - 3) \\ = x(5 + 5x - 7x^2) - 3(5 + 5x - 7x^2) \\ = \underline{5x} + \underline{5x^2} - \underline{7x^3} - \underline{15} - \underline{15x} + \underline{21x^2} \\ = -7x^3 + 26x^2 - 10x - 15 \end{aligned}$$

Let's look at the last section from yesterday's notes
Factor Completely:

$$\begin{aligned} 1. \quad & 36x^2 - 4y^2 \\ &= 4(9x^2 - y^2) \\ &= 4(3x - y)(3x + y) \end{aligned}$$

$$\begin{aligned} 2. \quad & 12x^2 - 27 \\ &= 3(4x^2 - 9) \\ &= 3(2x + 3)(2x - 3) \end{aligned}$$

$$\begin{aligned} 3. \quad & 2y^4 + 2y^3 + 4y^2 + 4y \quad \text{GCF} \\ &= 2y(y^3 + y^2 + 2y + 2) \\ &= 2y[y^2(y+1) + 2(y+1)] \\ &= 2y(y+1)(y^2 + 2) \end{aligned}$$

$$\begin{aligned} 4. \quad & 81 - m^4 \\ &= (9 - m^2)(9 + m^2) \\ &= (3 - m)(3 + m)(9 + m^2) \end{aligned}$$

YouTube video on RSA encryption (about 10 minutes)

(<http://www.youtube.com/watch?v=M7kEpw1tn50>)

<https://www.youtube.com/watch?v=M7kEpw1tn50>



1-5: Factoring Long and Short Product/Sum

 $a \neq 1$ Factoring Trinomials using Product/Sum (with leading coefficient $\neq 1$):

Example: Factor the following trinomials using Product/Sum long form.

$$\begin{array}{l}
 \overbrace{ax^2+bx+c} \\
 1. \ 2x^2+13x+6 \quad P=2(6)=12 \\
 \quad \quad \quad S=13 \\
 \quad \quad \quad \begin{array}{r} 2x^2+12x+1x+6 \\ 2x(x+6)+1(x+6) \\ (x+6)(2x+1) \end{array} \quad \begin{array}{l} 12, 1 \end{array}
 \end{array}$$

$$\begin{array}{l}
 \overbrace{6x^2-13x+5} \\
 2. \ 6x^2-13x+5 \quad P=30 \\
 \quad \quad \quad S=-13 \\
 \quad \quad \quad \begin{array}{r} 6x^2-3x-10x+5 \\ 3x(2x-1)-5(2x-1) \\ (2x-1)(3x-5) \end{array} \quad \begin{array}{r} 1 \ 30 \\ 2 \ 15 \\ -3 \ 10 \end{array}
 \end{array}$$

$$\begin{array}{l}
 \overbrace{4x^2+4x-3} \\
 3. \ 4x^2+4x-3 \quad P=-12 \\
 \quad \quad \quad S=4 \\
 \quad \quad \quad \begin{array}{r} 4x^2+2x+6x-3 \\ 2x(2x+1)+3(2x+1) \\ (2x+1)(2x+3) \end{array} \quad \begin{array}{r} 1 \ 12 \\ -2 \ 6 \end{array}
 \end{array}$$

$$\begin{array}{l}
 \overbrace{4x^2-4x-3} \\
 4. \ 4x^2-4x-3 \quad P=-12, S=-4 \\
 \quad \quad \quad \begin{array}{r} 4x^2+2x+6x-3 \\ 2x(2x+1)-3(2x+1) \\ (2x+1)(2x-3) \end{array} \quad \begin{array}{r} 1 \ 12 \\ 2 \ 6 \end{array}
 \end{array}$$

8. $5x^2 - 2x - 3$ $R = -15, S = -2$

$$\begin{array}{r} 5x^2 - 5x + 3x - 3 \\ \hline 5x(x-1) + 3(x-1) \\ (x-1)(5x+3) \end{array} \quad \begin{array}{l} 3, -5 \end{array}$$

Tip for finding pair of numbers that give correct product/sum:

Enter the following into Y_1 and Y_2 on calculator: $Y_1 = \text{prod}/x$

$$Y_2 = \text{prod}/x + x$$

Go to the table and find the sum in Y_2 column. The pair of numbers you want are the corresponding X and Y_1

Example: $x^2 - 19x + 48$

Prod = 48, Sum = -19

NORMAL FLOAT AUTO a+bi RADIAN MP			
Plot1	Plot2	Plot3	
$Y_1 = 48/X$			
$Y_2 = 48/X + X$			
$Y_3 =$			
$Y_4 =$			
$Y_5 =$			
$Y_6 =$			
$Y_7 =$			
$Y_8 =$			
$Y_9 =$			

NORMAL FLOAT AUTO a+bi RADIAN MP			
PRESS $\blacktriangle$ TO EDIT FUNCTION			
X	Y_1	Y_2	
-5	-9.6	-14.6	
-4	-12	-16	
-3	-16	-19	
-2	-24	-26	
-1	-48	-49	
0	ERROR	ERROR	
1	48	49	
2	24	26	
3	16	19	
4	12	16	
5	9.6	14.6	

$Y_2 = -19$

From the table you can see the correct numbers are -3, -16

Factoring Trinomials (with leading coefficient = 1):

Example: Factor the following trinomials using Product/Sum short cut.

$a=1$

$$1. \ x^2 + 6x + 8 \quad P=8, S=6$$

$$(x+4)(x+2) \quad 4, 2$$

$$2. \ x^2 - 13x + 40 \quad P=40, S=-13$$

$$= (x-8)(x-5) \quad -8, -5$$

$$3. \ x^2 - 3x - 40 \quad P=-40, S=-3$$

$$(x-8)(x+5) \quad -8, 5$$

$$4. \ x^2 + 3x - 10 \quad P=-10, S=3$$

$$(x+5)(x-2) \quad 5, -2$$

$$5. x^2 + 7x + 6 \quad P=6, S=7$$
$$= (x+6)(x+1)$$

$$6. x^2 + 5x + 6$$
$$(x+2)(x+3)$$

$$7. x^2 + 5x - 6$$
$$\cancel{(x-3)(x-2)}$$
$$(x-1)(x+6)$$

$$8. x^2 - 6x + 5$$
$$(x-5)(x-1)$$

Factor Completely: $2y^3 + 6y^2 + 4y$

$$2y(y^2 + 3y + 2)$$

$$= 2y(y + 1)(y + 2)$$

What is different about this one? Try factoring it.

$$x^2 + \underline{2xy} - 24y^2 \quad P = -24, S = 2$$

$$\underline{x^2 - 4xy + 6xy - 24y^2} \quad -4, 6$$

$$x(x - 4y) + 6y(x - 4y)$$

$$(x - 4y)(x + 6y)$$

