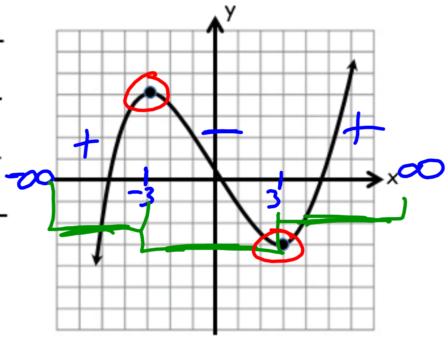
Quiz 4

1. Given the graph at right, determine each of the following:

x2 x-intervals
Increasing: (-∞, -3) (3, ∞)
Decreasing: (-3, 3)

x2
Relative Min: (3, -3)
Relative Max: (-3, 4)



Nov 18-4:49 PM

2. Find the zeros of the polynomial, state the multiplicity of each. Sketch (including the end behavior)

$P(x) = x(x-2)^2(x+1)$

$x \cdot x^2 \cdot x = x^4$

Degree: 4

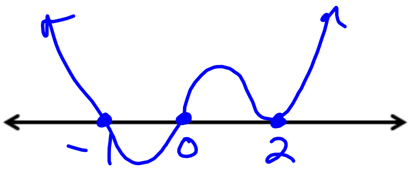
$+x^{even}$

End Behavior: ↖ ↗

zeros, multiplicity, tangent (m = even), cross (m = odd)

Z	M	T/C
0	1	C
2	2	T
-1	1	C

Sketch:



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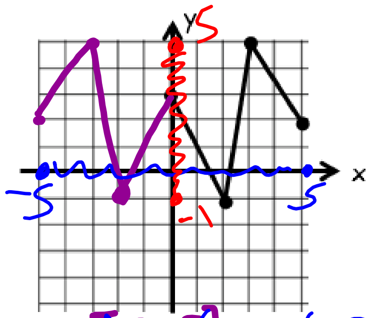
Given the **partial graph** of function f shown below. Sketch the other half of the function in a) if f(x) is **even** and in b) if f(x) is **odd**. Find f(-x) for each of the indicated points. State the domain and range of each completed function.

HW 5-10

4

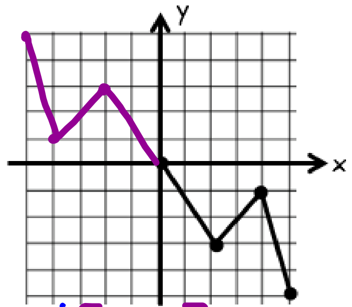
1. Even



Domain: [-5,5] (-5, 5)

Range: ~~[-1,5]~~

2. Odd



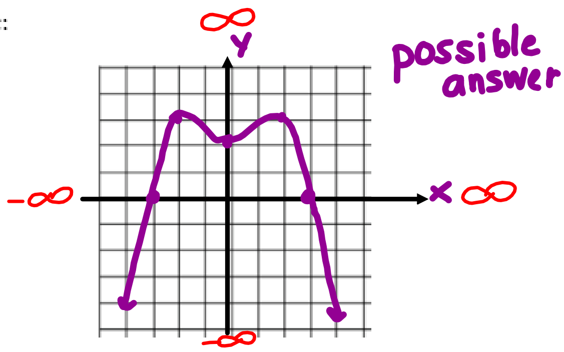
Domain: ~~[-5,5]~~

Range: ~~[-5,5]~~

Nov 2-12:18 PM

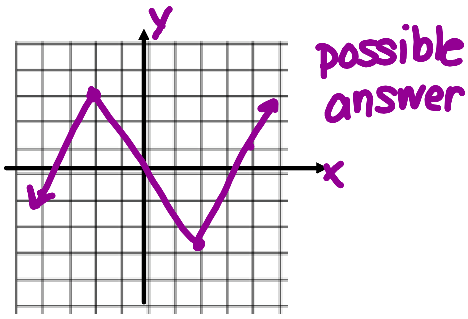
3. Draw a function with the following properties:

a. Zeros of 3 and -3
b. As x → ∞ y → -∞
c. The graph is even



4. Draw a function with the following properties:

a. It passes through the point (-2, 3)
b. As x → -∞ y → -∞
c. The graph is odd



Nov 2-12:19 PM

Given the partially filled out table below for f(x), fill in the rest of the table based on the function type.

$x > y$
$-x$

$x \rightarrow y$
$-x \rightarrow -y$

7. Even

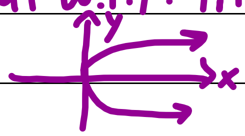
x	-3	-2	-1	0	1	2	3
y	17	7	1	-1	1	7	17

8. Odd

x	-3	-2	-1	0	1	2	3
y	18	2	-2	0	+2	-2	-18

9. Even functions have symmetry across the y-axis. Odd functions have symmetry across the origin. Can a function have symmetry across the x-axis? Why or why not?

If a graph is symmetrical w.r.t. the x-axis it's not a funtion. i.e.



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$f(-x) =$ same $\rightarrow$ even
$=$ opposite $\rightarrow$ odd
otherwise

Determine algebraically whether each of the following functions is even, odd, or neither. Check your answer using tables on your graphing calculator.

10. $f(x) = 2x + 1$

$f(-x) = 2(-x) + 1$
$= -2x + 1$

not same
not opp
$\therefore$ neither

11. $f(x) = 2x^2 + 1$

x^2

$f(-x) = 2(-x)^2 + 1$
$= 2x^2 + 1$

same $\rightarrow$ even

12. $f(x) = 2x^3 + x$

$f(-x) = 2(-x)^3 + (-x)$
$= -2x^3 - x$

opp $\rightarrow$ odd

$f(x) = \dfrac{1}{2x+1}$

opposite $\dfrac{1}{-2x-1}$ or $\dfrac{-1}{2x+1}$

$f(-x) = \dfrac{1}{-2x+1}$

opposite

$\dfrac{1}{2} \rightarrow \boxed{-\dfrac{1}{2} = \dfrac{-1}{2} = \dfrac{1}{-2}} \neq \dfrac{-1}{-2}$

Nov 2-12:19 PM

13. The cubic $x^3 + 7x^2 + 13x + 3$ has only one rational zero, $x = -3$.
Use polynomial long division to show that the remainder is zero when dividing the cubic by $x + 3$.
Then use the quadratic formula to find the other two (irrational) zeros.

$$\begin{array}{r} x^2 + 4x + 1 \\ x+3 \overline{) x^3 + 7x^2 + 13x + 3} \\ -x^3 - 3x^2 \quad\quad\quad \\ \hline 4x^2 + 13x \quad \\ -4x^2 - 12x \quad \\ \hline x + 3 \\ -x - 3 \\ \hline 0 \end{array}$$

Aside:
$x^2(x+3) = x^3 + 3x^2$
$4x(x+3) = 4x^2 + 12x$

$a=1,\ b=4,\ c=1$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{12}}{2} = \frac{-4 \pm \sqrt{4}\sqrt{3}}{2} = \frac{-4 \pm 2\sqrt{3}}{2} = \frac{-2 \pm \sqrt{3}}{\cancel{1}}$$

$x^2 + 4x + 1 = 0$
$b^2 - 4ac = 16 - 4(1)(1) = 12$
$\sqrt{12} = 2\sqrt{3}$
$$x = \frac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3}$$

Oct 22-5:21 PM

Return Groupwork

With your group, complete the following problem. This was from the June '19 Algebra II Regents Exam.

Factor completely over the set of integers: $16x^4 - 81$

$= (4x^2-9)(4x^2+9)$
$= (2x+3)(2x-3)(4x^2+9)$

Sara graphed the polynomial $y = 16x^4 - 81$ and stated "All roots of $y = 16x^4 - 81$ are real."

Is Sara correct? Explain your reasoning. No, Sara is not correct.
Graphically the function crosses the x-axis twice, at ± 1.5. Those zeros are consistent w/ factors $2x \pm 3$ above. But $4x^2+9$ results in 2 imaginary zeros at $\pm 1.5i$.

$4x^2 + 9 = 0$
$\sqrt{x^2} = \sqrt{-\frac{9}{4}}$
$x = \pm\frac{3i}{2}$ or $\pm 1.5i$

Nov 17-1:06 PM

Answers on last 2 pages

1. Sketch: $y = 2x^3 + 4x^2 - 3$

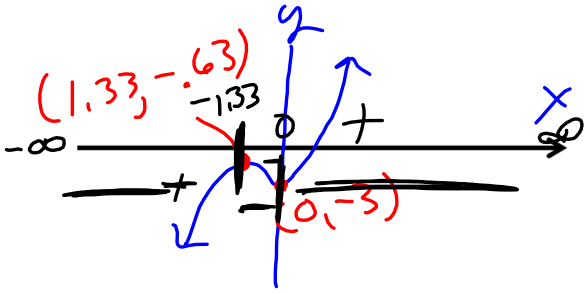
Find:

Increasing: $(-\infty, -1.33)$ $(0, \infty)$

Decreasing: $(-1.33, 0)$

Rel Min: $(0, -3)$

Rel Max: $(-1.33, -.63)$



$x = \cancel{8.2372E-7}\ 0$

$y = -3$

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2. Find the zeros of the polynomial, state the multiplicity of each. Sketch (including the end behavior)

Review Key is at the end of this file

$P(x) = x(x + 3)^2(x - 1)$

Z	M	T/C

Degree: ________

End Behavior:

Sketch:

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3 – 6: Factor completely

3. $3x^3 + 6x^2 - x - 2$

4. $x^{4n} - 5x^{2n} + 4$

5. $64x^3 + 27$

6. $(x - 3)^2 - (x - 3) - 6$

Oct 29-1:25 PM

7 – 10: Solve (Factor completely first)

7. $x^4 - 40x^2 + 144 = 0$

8. $2x^3 - 3x^2 - 10x + 15 = 0$

9. $x^5 - 81x = 0$

10. $3(x - 1)^2 - (x - 1) - 2 = 0$

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11. Use long division to find the quotient (Q(x)) and remainder (R(x)). Verify your remainder with the remainder theorem.

$(2x^3 + 5x^2 + 3x - 4) \div (x + 2)$

Is $(x + 2)$ a factor of $2x^3 + 5x^2 + 3x - 4$? Explain your answer.

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12. Given the polynomial $P(x) = x^3 + x^2 + kx - 4$, find the value of k such that $x - 2$ is a factor of P.

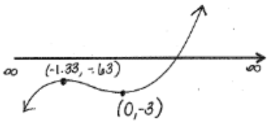
Using your value of k, find the other factors of P. (either sketch or algebraically)

Oct 29-1:26 PM

Name Key
Date ______

Alg2CC
Review Unit 5

1. Sketch: $y = 2x^3 + 4x^2 - 3$
Find:
Increasing: $(-\infty, -1.33), (0, \infty)$
Decreasing: $(-1.33, 0)$
Rel Min: $(0, -3)$
Rel Max: $(-1.33, -.63)$



2. Find the zeros of the polynomial, state the multiplicity of each. Sketch (including the end behavior)

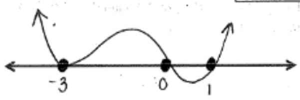
$P(x) = x(x + 3)^2(x - 1)$

Degree: 4

End Behavior: ↑ ↑

Z	M	T/C
-3	2	T
0	1	C
1	1	C

Sketch:



3 - 6: Factor completely

3. $3x^3 + 6x^2 - x - 2$
$= 3x^2(x+2) - 1(x+2)$
$= (3x^2-1)(x+2)$

4. $x^{4n} - 5x^{2n} + 4$
$= (x^{2n}-4)(x^{2n}-1)$
$= (x^n-2)(x^n+2)(x^n-1)(x^n+1)$

5. $64x^3 + 27 = (4x+3)(16x^2-12x+9)$
$a = 4x$
$b = 3$

6. $(x - 3)^2 - (x - 3) - 6$ Let $u = x-3$
$u^2 - u - 6$
$(u-3)(u+2)$
$(x-3-3)(x-3+2)$
$(x-6)(x-1)$

7 - 10: Solve (Factor completely first)

7. $x^4 - 40x^2 + 144 = 0$
$(x^2-4)(x^2-36) = 0$
$(x-2)(x+2)(x-6)(x+6) = 0$
$x=2 \quad x=-2 \quad x=6 \quad x=-6$
$\{\pm 2, \pm 6\}$

8. $2x^3 - 3x^2 - 10x + 15 = 0$
$x^2(2x-3) - 5(2x-3) = 0$
$(2x-3)(x^2-5) = 0$ $\{\frac{3}{2}, \pm\sqrt{5}\}$
$2x-3=0 \quad x^2-5=0$
$x = \frac{3}{2} \quad x^2 = 5$
$x = \pm\sqrt{5}$

9. $x^5 - 81x = 0$ $\{0, \pm 3i, \pm 3\}$
$x(x^4-9) = 0$
$x(x^2+9)(x^2-9) = 0$
$x(x^2+9)(x+3)(x-3) = 0$
$x=0 \quad x^2+9=0 \quad x+3=0 \quad x-3=0$
$x^2 = -9 \quad x=-3 \quad x=3$
$x = \pm 3i$

10. $3(x - 1)^2 - (x - 1) - 2 = 0$ Let $u = x-1$
$3u^2 - u - 2 = 0$
$3u^2 - 3u + 2u - 2 = 0$
$3u(u-1) + 2(u-1) = 0$
$(3u+2)(u-1) = 0$
$(3(x-1)+2)(x-1-1) = 0$
$(3x-1)(x-2) = 0$
$3x-1=0$
$x = \frac{1}{3}$
$x-2=0$
$x=2$
$\{\frac{1}{3}, 2\}$

11. Use long division to find the quotient (Q(x)) and remainder (R(x)). Verify your remainder with the remainder theorem.

$(2x^3 + 5x^2 + 3x - 4) \div (x + 2)$

$$\begin{array}{r} 2x^2 + x + 1 \\ x+2 \overline{)\,2x^3+5x^2+3x-4} \\ -2x^3-4x^2 \\ \hline x^2+3x \\ -x^2-2x \\ \hline x-4 \\ -x-2 \\ \hline -6 \end{array}$$

Aside:
$2x^2(x+2) = 2x^3 + 4x^2$
$x(x+2) = x^2 + 2x$
$1(x+2) = x+2$

$\boxed{Q(x) = 2x^2 + x + 1 \quad R(x) = -6}$

$P(-2) = 2(-2)^3 + 5(-2)^2 + 3(-2) - 4$
$P(-2) = -6$ ✓

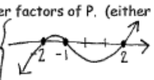
Is (x + 2) a factor of $2x^3 + 5x^2 + 3x - 4$? Explain your answer.
No. If $x+2$ was a factor, the remainder would be 0.

12. Given the polynomial $P(x) = x^3 + x^2 + kx - 4$, find the value of k such that $x - 2$ is a factor of P.
$P(2) = 2^3 + 2^2 + 2k - 4$
$0 = 8 + 2k$
$2k = -8$
$k = -4$
$P(x) = x^3 + x^2 - 4x - 4$

Using your value of k, find the other factors of P. (either sketch or algebraically)

Algebraically
$P(x) = x^2(x+1) - 4(x+1)$
$P(x) = (x+1)(x^2-4)$
$P(x) = (x+1)(x-2)(x+2)$

Graphically



$P(x) = (x+2)(x+1)(x-2)$
∴ other factors: $(x+1)(x+2)$

Nov 18-11:16 AM